

Challenges and progress toward dynamical stabilization of advanced bosonic qubits

Philippe Campagne-Ibarcq

Quantic team, ENS-PSL, CNRS, Inria, Mines Paris, Université PSL

Brieuc Beauseigneur
Louis Lattier
Lev-Arcady Sellem
Aron Vanselow

Zaki Leghtas
Mazyar Mirrahimi
Rémi Robin
Pierre Rouchon
Alain Sarlette

Inria



Back in 2019...



Do you want to participate in Q-feedback ?

Of course, I would love some support on my GKP project !



I'll send you the draft to check the GKP part

The scientific part is great, but why are you requesting 1.9 million euros only ?



I don't really know how to spend more...

... Have you ever thought about buying a dilution cryostat ?

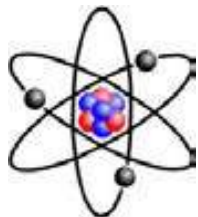


And in 2021...

“ Pierre, meet your fridge, Boris ! ”



Bits and qubits



0



1

Bits and qubits



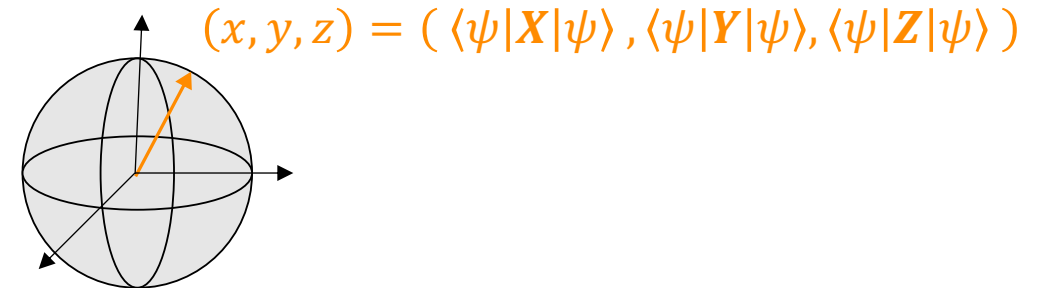
1 qubit state = normalized vector in 2D Hilbert space

$$|\psi\rangle = \alpha|0\rangle + e^{i\theta}\sqrt{1-\alpha}|1\rangle$$

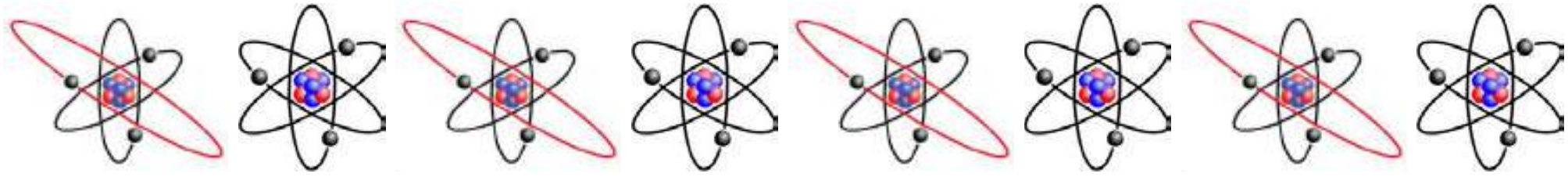
Pauli matrices:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Bloch sphere representation:



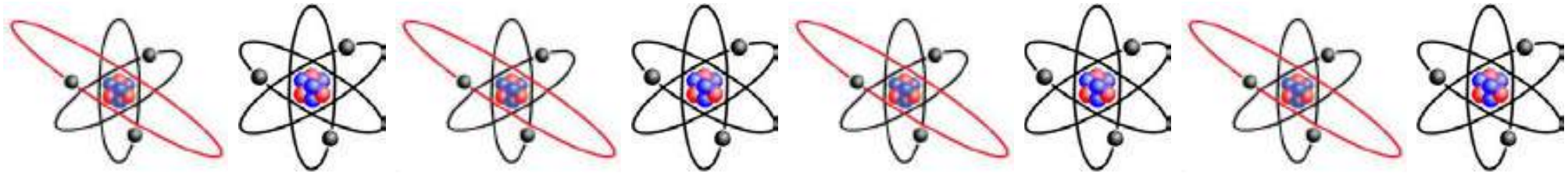
Bits and qubits



$$\alpha_0|000000\rangle + e^{i\theta_1}\alpha_1|000001\rangle + e^{i\theta_2}\alpha_2|000010\rangle + \dots$$

N qubits state = normalized vector in 2^N - D Hilbert space

Bits and qubits

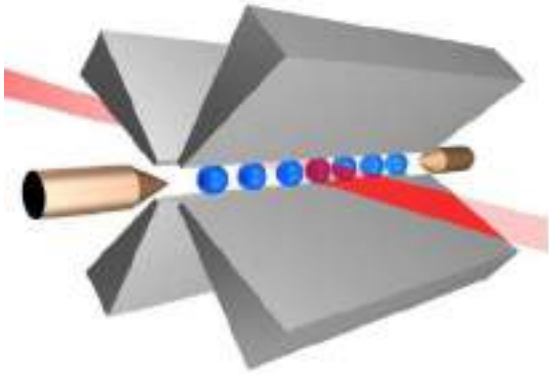


$$\alpha_0|000000\rangle + e^{i\theta_1}\alpha_1|000001\rangle + e^{i\theta_2}\alpha_2|000010\rangle + \dots$$

N qubits state = normalized vector in 2^N - D Hilbert space

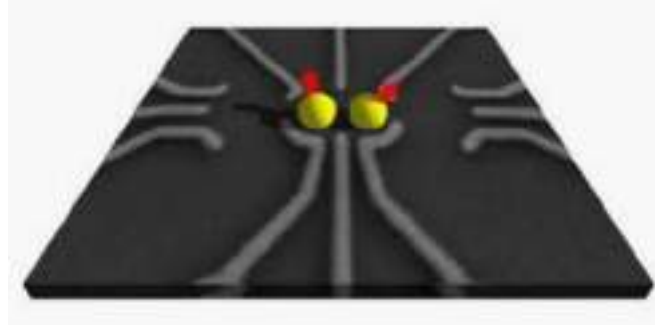
Quantum computation = unitary transformation = physical evolution

Physical platforms



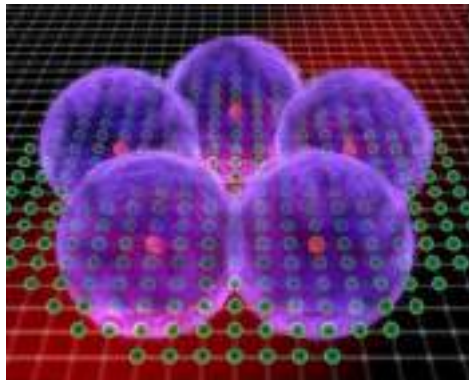
Trapped ions

(picture : Innsbruck university)



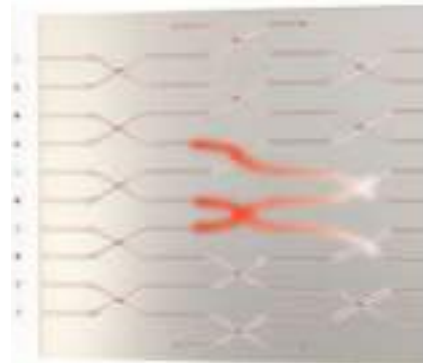
Spins in semiconductor

(picture : Basel university)



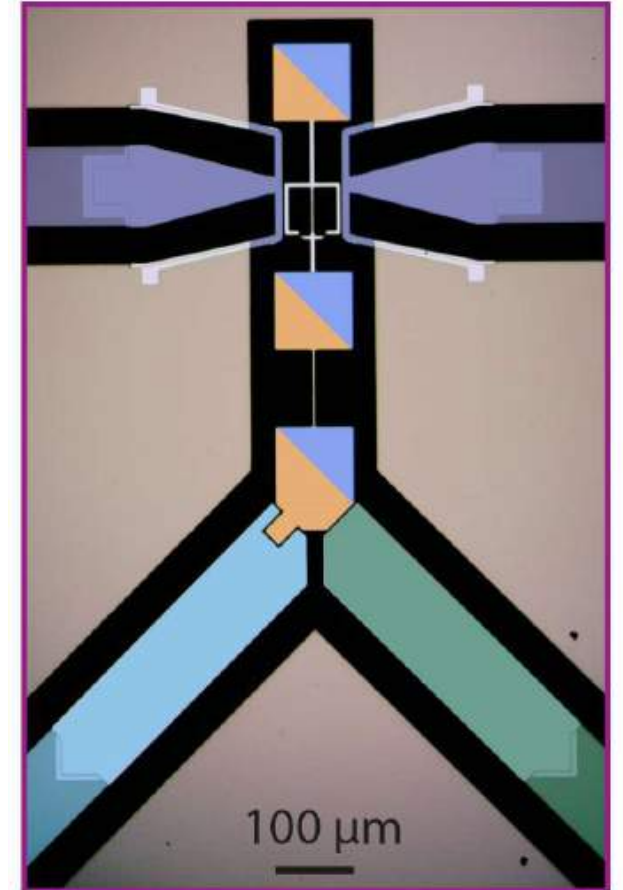
Rydberg atoms

(picture : Max Planck institute)



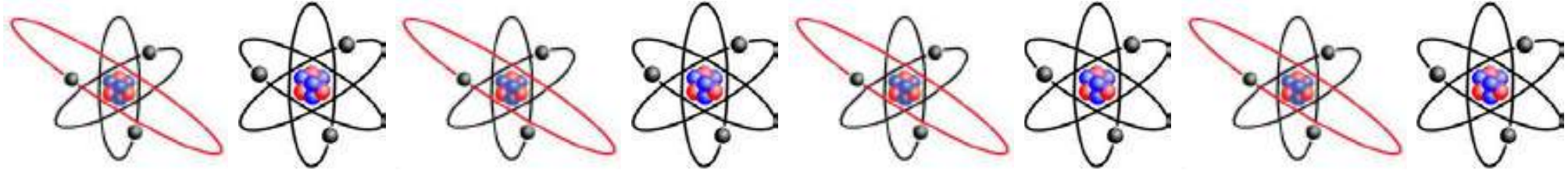
Photonic chips

(picture : Oxford)



Superconducting circuits

Noise

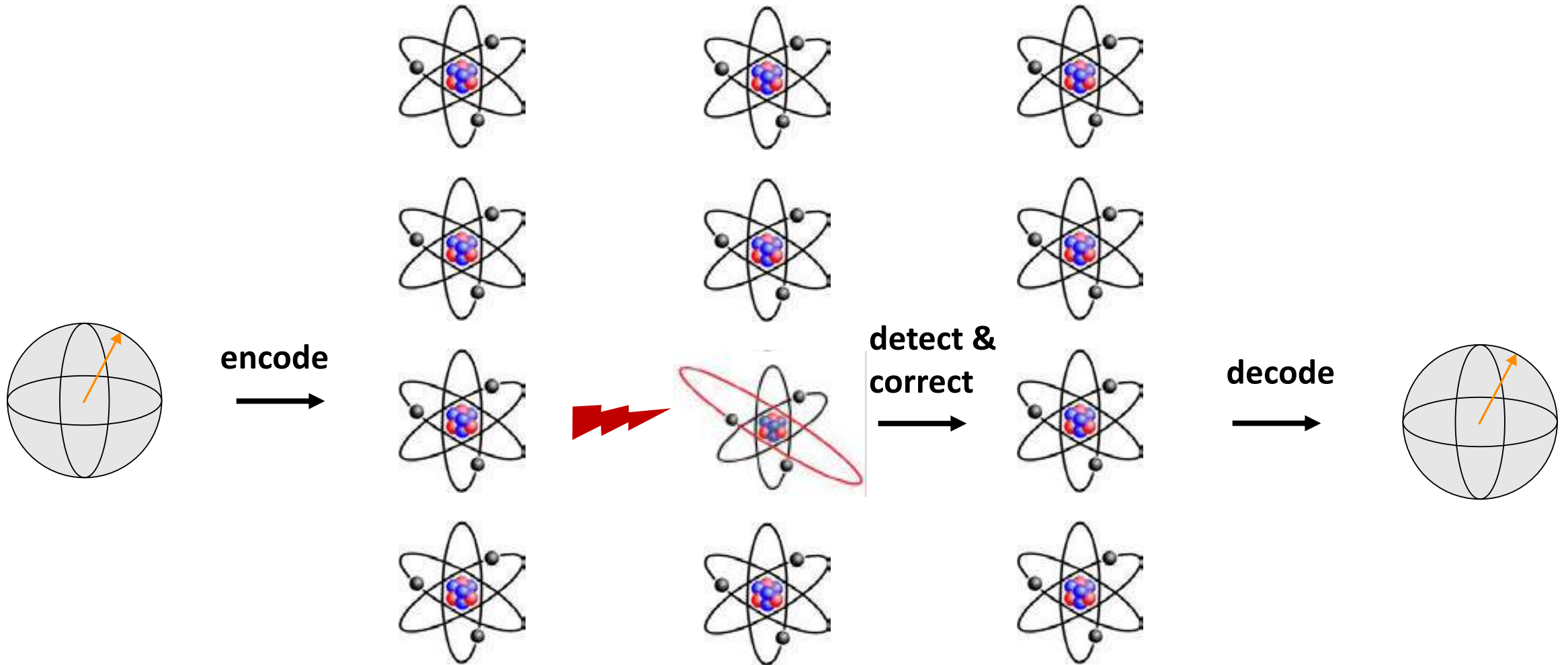


$$\alpha_0|000000\rangle + e^{i\tilde{\theta}_1}\alpha_1|000001\rangle + e^{i\theta_2}\tilde{\alpha}_2|000010\rangle + \dots$$

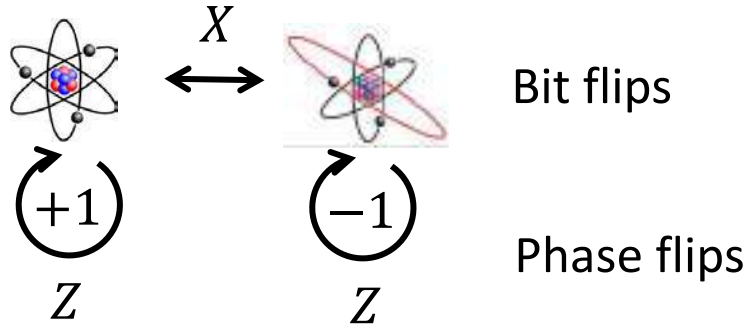


Coupling to environment → stochastic evolution

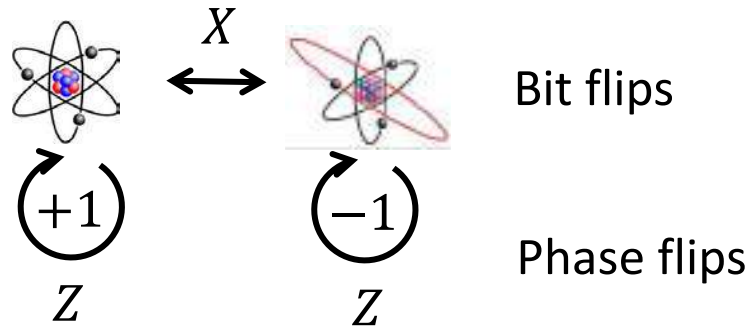
Quantum error-correction



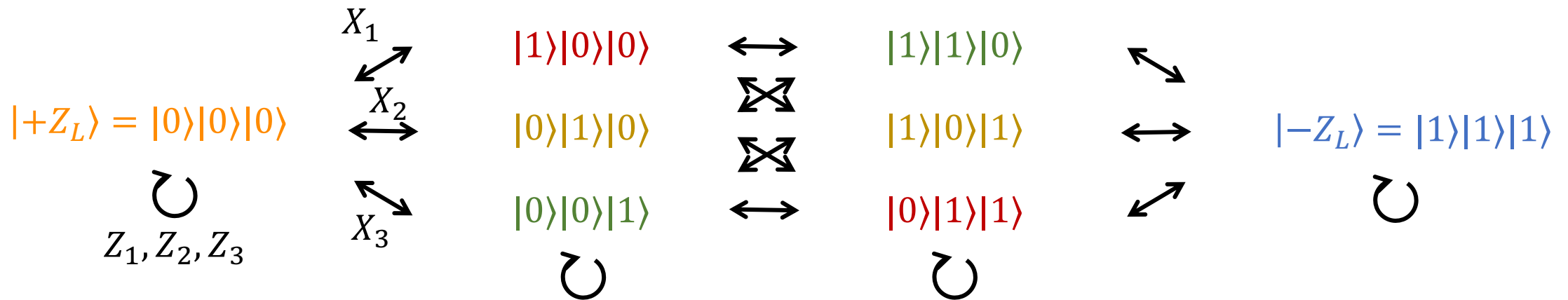
Bit flips and phase flips



Discrete variable codes: repetition code

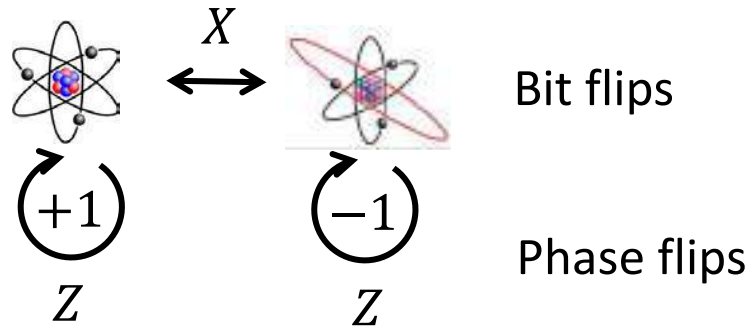


Noise is local (uncorrelated errors)



Stabilizers: $\{Z_1Z_2, Z_2Z_3, Z_3Z_1\}$

Discrete variable codes: repetition code

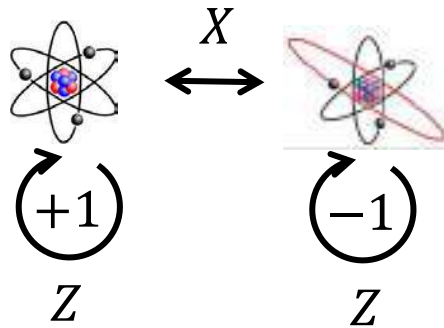


Noise is local (uncorrelated errors)

$$|+X_L\rangle = \frac{|+Z_L\rangle + |-Z_L\rangle}{\sqrt{2}} \quad \xleftrightarrow{Z_1, Z_2, Z_3} \quad |-X_L\rangle = \frac{|+Z_L\rangle - |-Z_L\rangle}{\sqrt{2}}$$

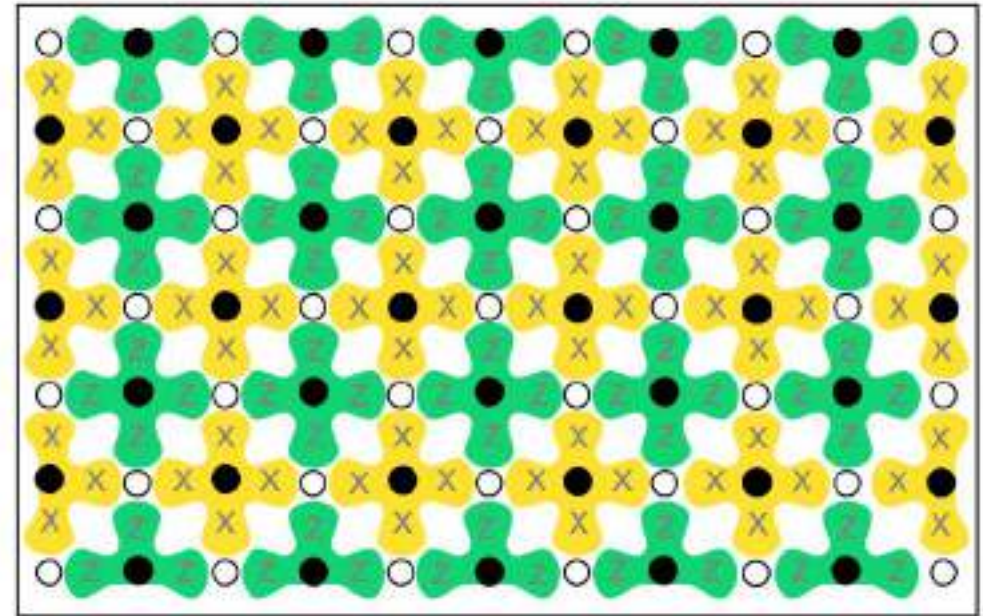
Discrete variable codes: surface code

Local noise:



Bit flips

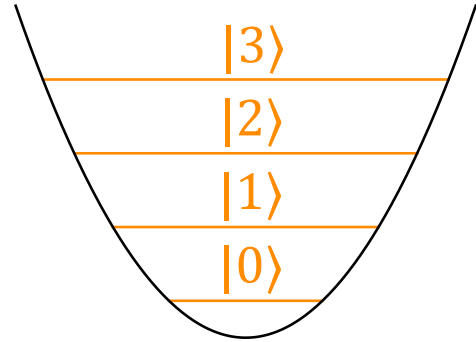
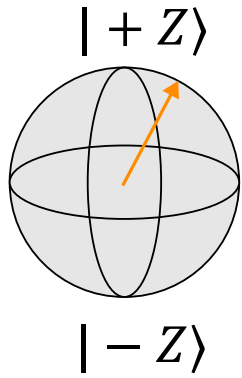
Phase flips



Fowler *et al.*, PRA 2012

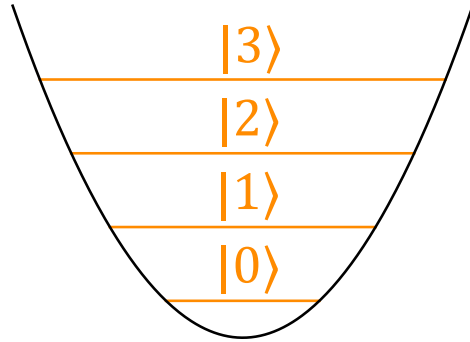
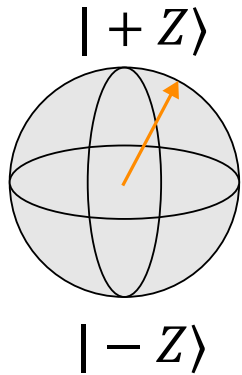
Stabilizers: $\{Z_i Z_j Z_k Z_l, X_{i'} X_{j'} X_{k'} X_{l'}\}_{i,j,k,l \text{ and } i',j',k',l' \text{ adjacent}}$

Bosonic qubits



Harmonic oscillator

The harmonic oscillator

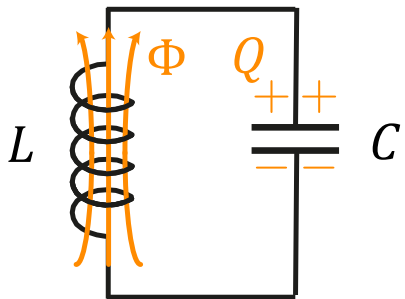


Harmonic oscillator

Dynamics (Schrödinger picture):

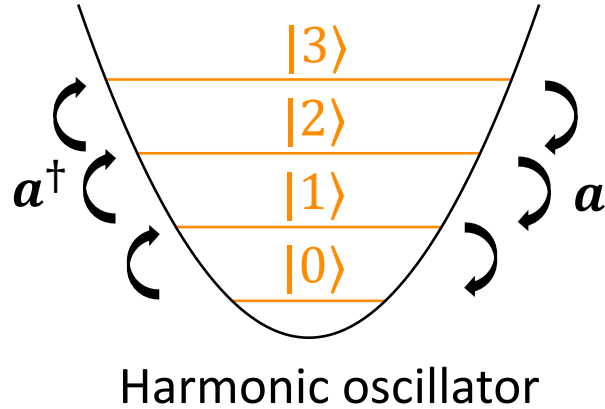
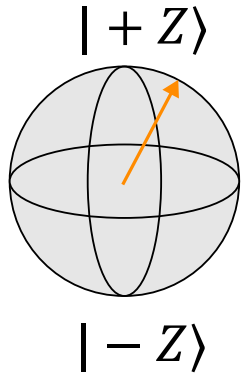
$$H_0 = \frac{1}{2C} Q^2 + \frac{1}{2L} \Phi^2$$

$$|\psi(t)\rangle = e^{-iH_0 t/\hbar} |\psi(0)\rangle$$



$$[\Phi, Q] = i\hbar$$

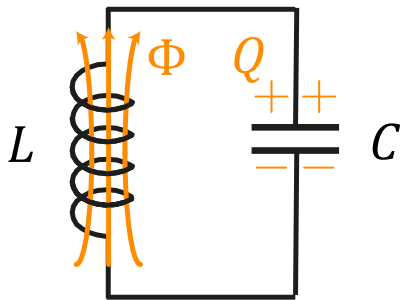
The harmonic oscillator



Dynamics (Schrödinger picture):

$$H_0 = \frac{1}{2} \omega (q_0^2 + p_0^2)$$

$$|\psi_0(t)\rangle = e^{-iH_0 t/\hbar} |\psi_0(t=0)\rangle$$



$$\omega = 1/\sqrt{LC}$$

$$Z = \sqrt{L/C}$$

$$q_0 = (\hbar Z)^{-1/2} \Phi$$

$$p_0 = (Z/\hbar)^{1/2} Q$$

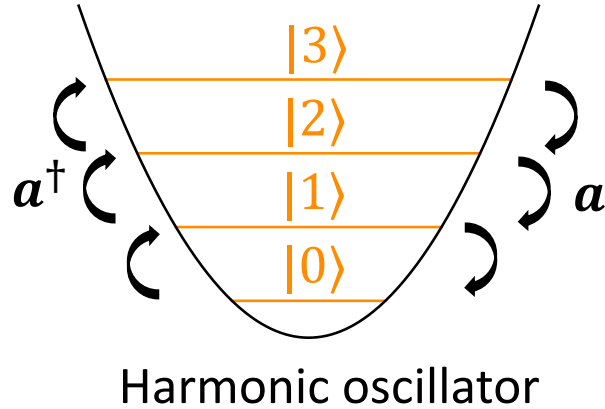
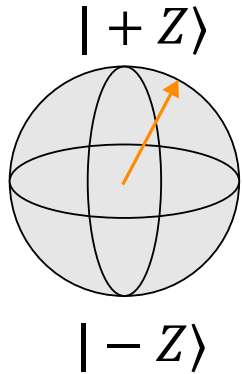
$$a = (q_0 + ip_0)/\sqrt{2}$$

$$a^\dagger = (q_0 - ip_0)/\sqrt{2}$$

$$[q_0, p_0] = i$$

$$[a, a^\dagger] = 1$$

The harmonic oscillator

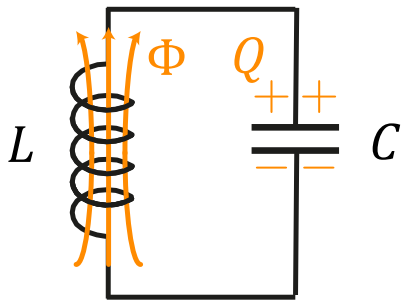


Dynamics (interaction picture):

$$|\psi(t)\rangle = |\psi(t=0)\rangle \quad (\mathbf{H} = 0)$$

$$\mathbf{q} = e^{\frac{iH_0 t}{\hbar}} \mathbf{q}_0 e^{-\frac{iH_0 t}{\hbar}}$$

$$\mathbf{p} = e^{\frac{iH_0 t}{\hbar}} \mathbf{p}_0 e^{-\frac{iH_0 t}{\hbar}}$$



$$\omega = 1/\sqrt{LC}$$

$$Z = \sqrt{L/C}$$

$$\mathbf{q}_0 = (\hbar Z)^{-1/2} \Phi$$

$$\mathbf{p}_0 = (Z/\hbar)^{1/2} Q$$

$$\mathbf{a} = (\mathbf{q}_0 + i\mathbf{p}_0)/\sqrt{2}$$

$$\mathbf{a}^\dagger = (\mathbf{q}_0 - i\mathbf{p}_0)/\sqrt{2}$$

$$[\mathbf{q}_0, \mathbf{p}_0] = i$$

$$[\mathbf{a}, \mathbf{a}^\dagger] = 1$$

State representation

Wavefunction

$$\forall r \in \mathbb{R}, \quad \mathbf{q}|r\rangle_q = r|r\rangle_q \\ \mathbf{p}|r\rangle_p = r|r\rangle_p$$

$$\psi(r) = {}_q\langle r|\psi\rangle$$

$$\tilde{\psi}(r) = {}_p\langle r|\psi\rangle$$

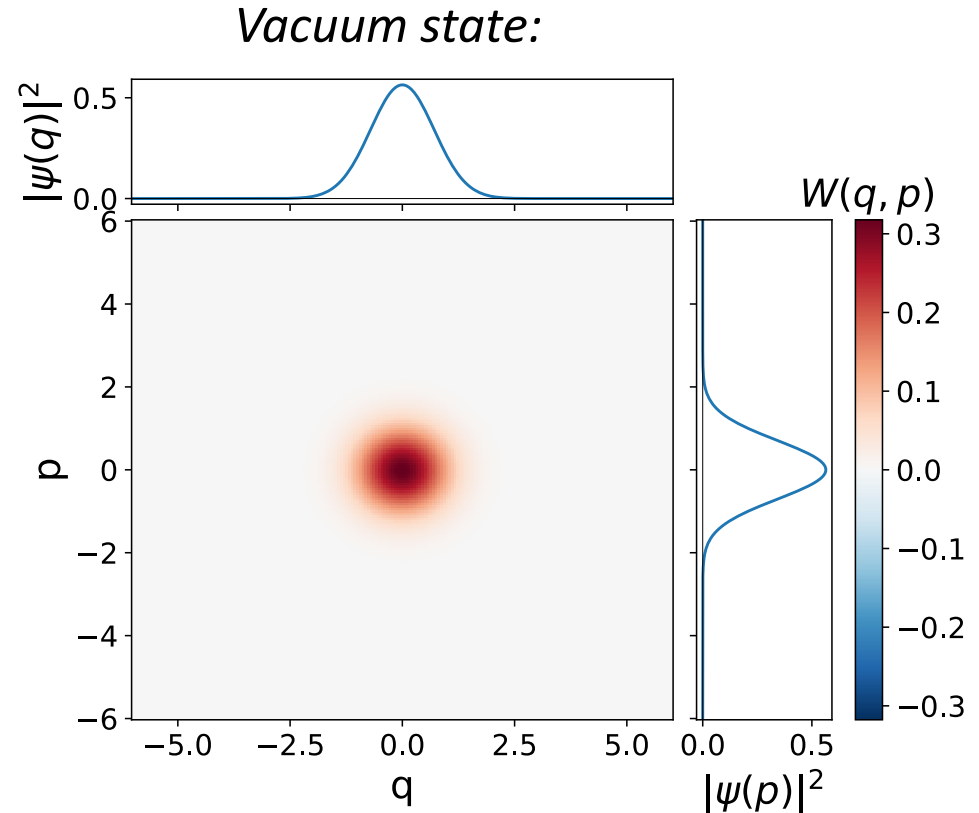
Density matrix

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

$$\sum_i p_i = 1$$

Wigner distribution

$$W(q, p) = \frac{1}{\pi} \int_{-\infty}^{\infty} \psi^*(q+r)\psi(q-r)e^{2ipr} dr$$



State representation

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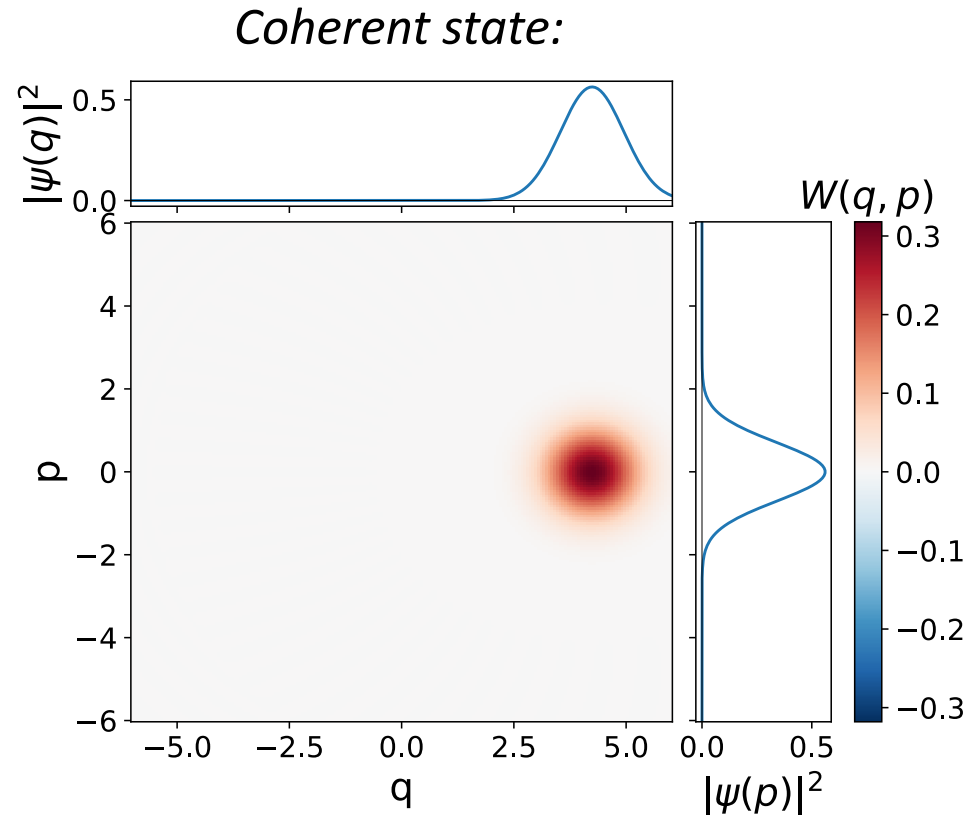
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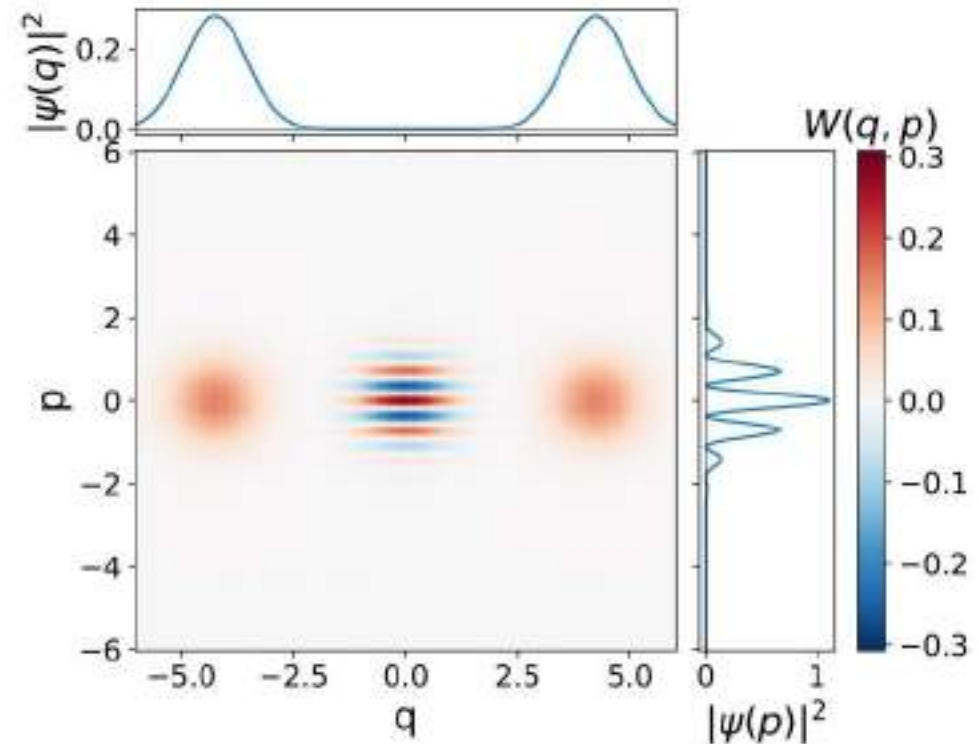
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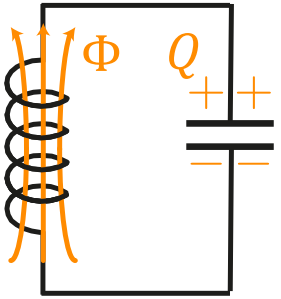
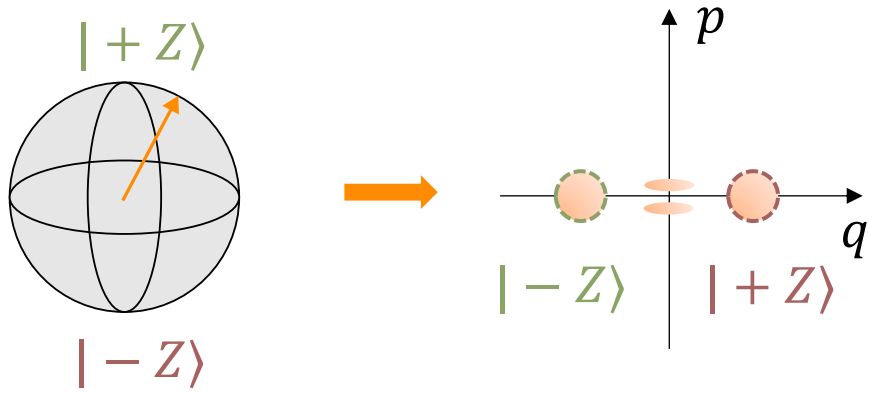
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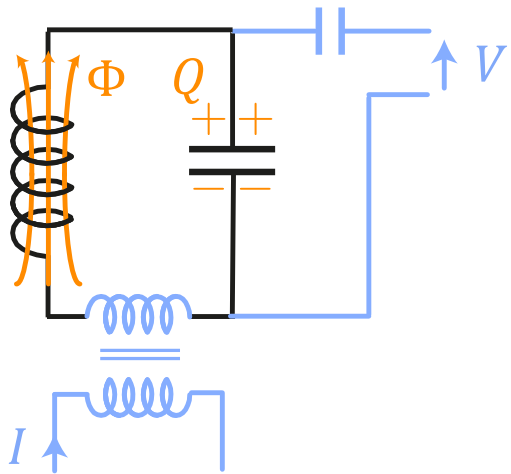
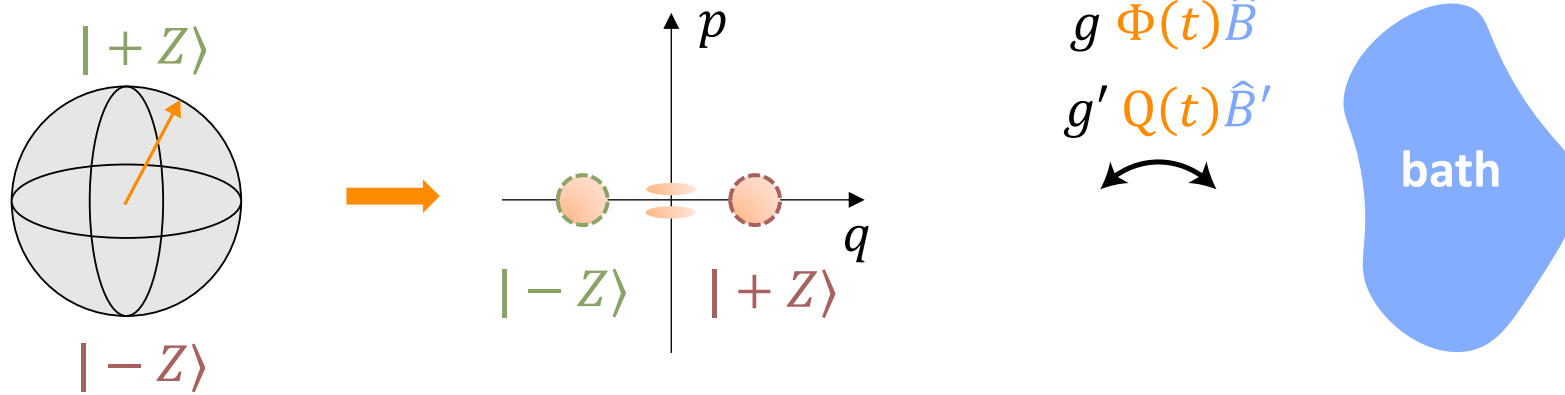
Cat state:



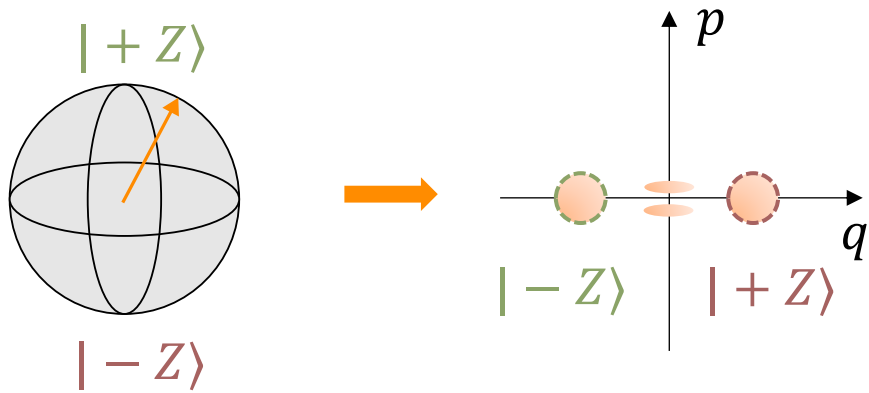
Noise structure



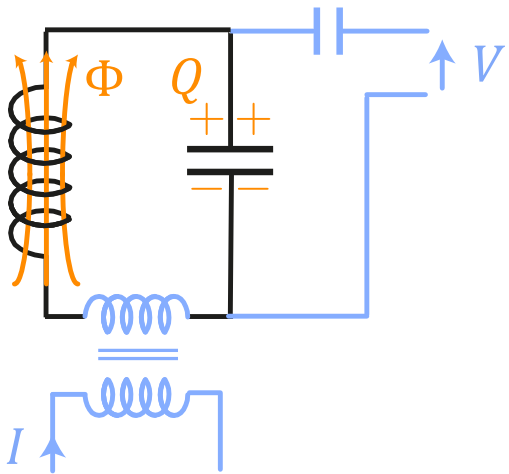
Noise structure



Noise structure



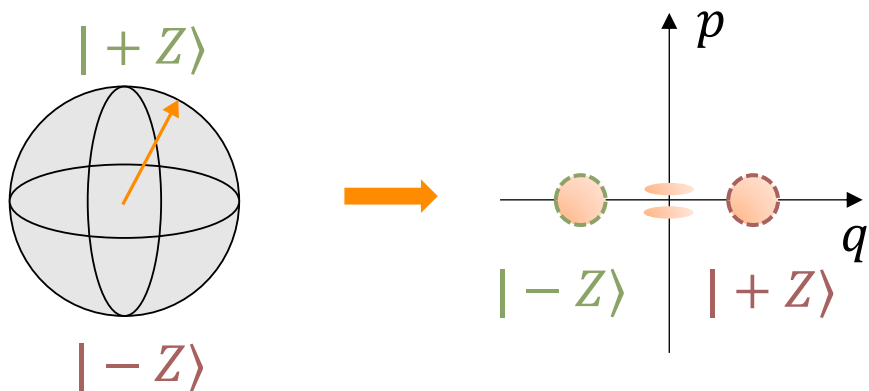
$$\begin{array}{c}
 g \Phi(t) \hat{B} \\
 g' Q(t) \hat{B}'
 \end{array}
 \begin{array}{c}
 \longleftrightarrow \\
 S_{BB}(\pm\omega) \\
 S_{B'B'}(\pm\omega)
 \end{array}
 \longrightarrow
 \begin{array}{c}
 \kappa (1 + n_{th}) \mathcal{D}[a] \\
 \kappa n_{th} \mathcal{D}[a^\dagger]
 \end{array}$$



$$\dot{\rho} = \sum_i \mathcal{D}[L_i](\rho) = \sum_i L_i \rho L_i^\dagger - \frac{1}{2} (L_i^\dagger L_i \rho + \rho L_i^\dagger L_i)$$

Noise in non-linear circuits

Diffusive evolution



$$g \Phi(t) \hat{B}$$

$$g' Q(t) \hat{B}'$$

$$S_{BB}(\pm\omega)$$

$$S_{B'B'}(\pm\omega)$$

Low-order dissipators :

$$\kappa (1 + n_{th}) \mathcal{D}[a]$$

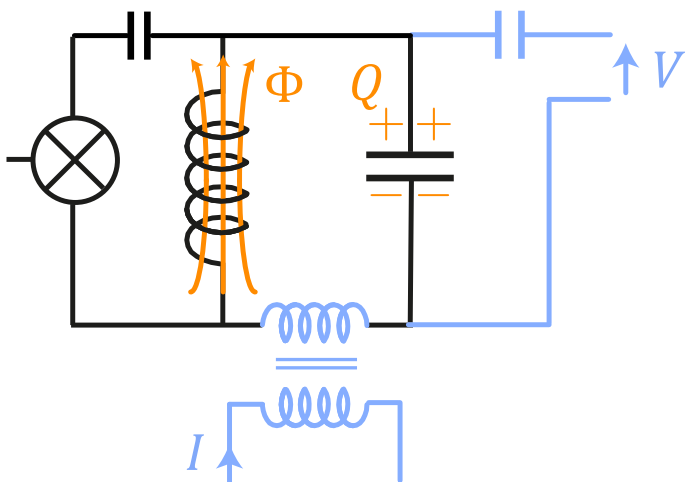
$$\kappa n_{th} \mathcal{D}[a^\dagger]$$

$$2\kappa_\phi \mathcal{D}[a^\dagger a]$$

$$\dots$$

$$H_c(t) = f(\Phi, t) = \sum_{n,k} f_{n,k}(t) (a^\dagger a)^k a^n + h.c.$$

Low-order non-linearity : $f_{n,k} \xrightarrow{n,k \rightarrow \infty} 0$

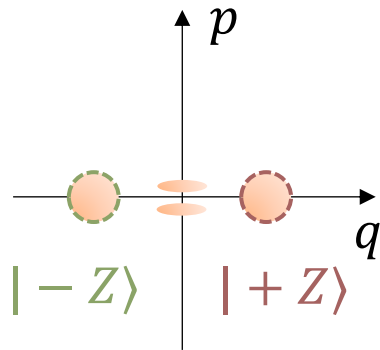


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Noise in non-linear circuits

Diffusive evolution

Distant states
→
no flip at short time



$$g \Phi(t) \hat{B}$$

$$g' Q(t) \hat{B}'$$

$$S_{BB}(\pm\omega)$$

$$S_{B'B'}(\pm\omega)$$

Low-order dissipators :

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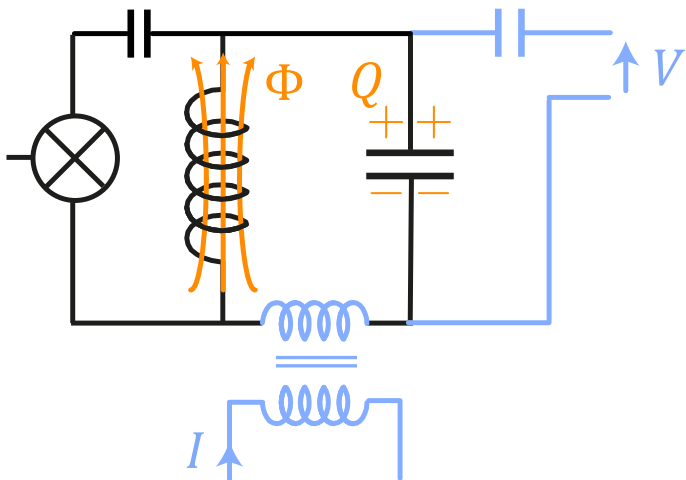
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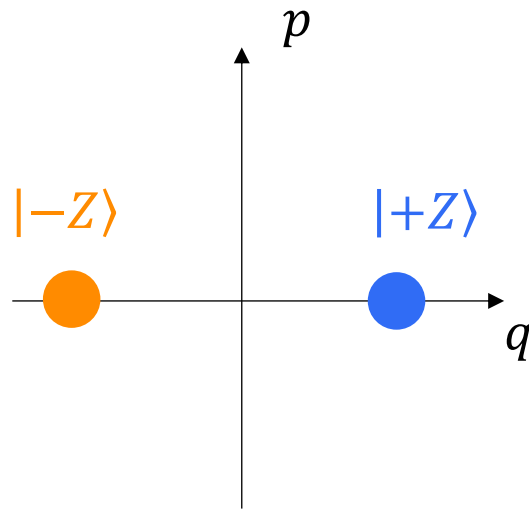
Outline

1. Bosonic codes: structure
2. Bosonic codes: dissipative stabilization
3. First experimental steps

Schrödinger cat qubits

Code : $\text{Span}\{|+\alpha\rangle, |-\alpha\rangle\}$

Stabilizer: $S = a^2 - \alpha^2 + 1$



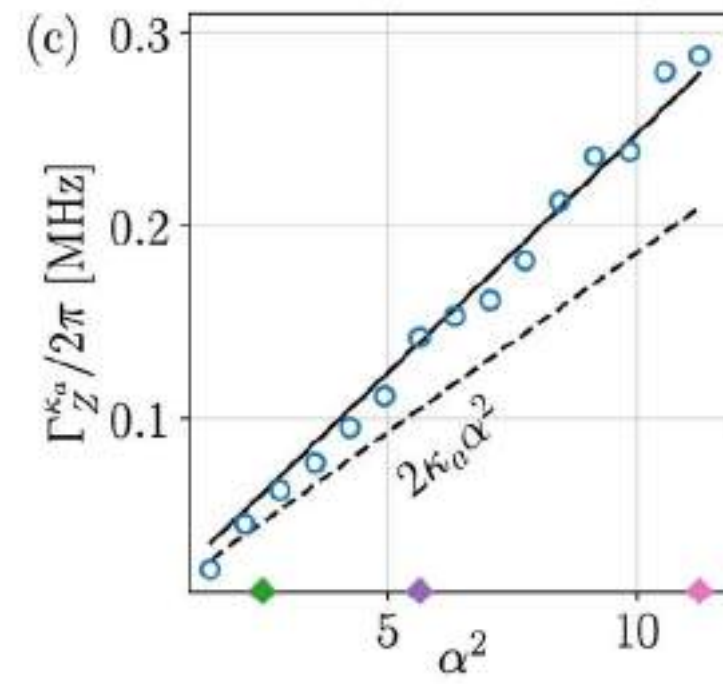
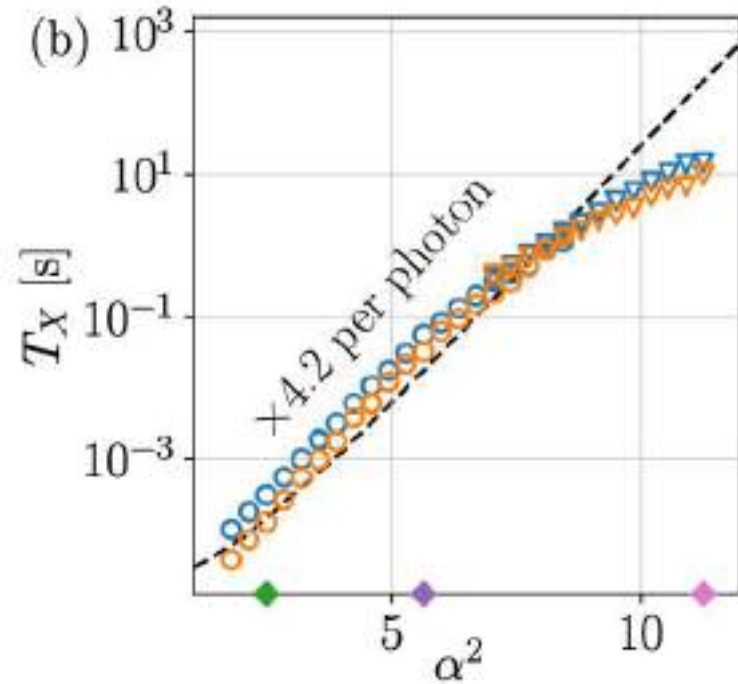
$$|+X\rangle = |\mathcal{C}_\alpha^+\rangle \sim |+\alpha\rangle + |-\alpha\rangle \quad \xleftrightarrow{a} \quad |-X\rangle = |\mathcal{C}_\alpha^-\rangle \sim |+\alpha\rangle - |-\alpha\rangle$$

(Even) (Odd)

$$|\pm\alpha\rangle \sim \sum_n \frac{(\pm\alpha)^n}{\sqrt{n!}} |n\rangle$$

$$a|\pm\alpha\rangle = \pm\alpha|\pm\alpha\rangle$$

A half-protected qubit



protected

$$|+Z\rangle = |\alpha\rangle \longleftrightarrow |-Z\rangle = |-\alpha\rangle$$

$$|+X\rangle = |\mathcal{C}_\alpha^+\rangle \overset{a}{\longleftrightarrow} |-X\rangle = |\mathcal{C}_\alpha^-\rangle$$

(Even) (Odd)

Repetition cat code

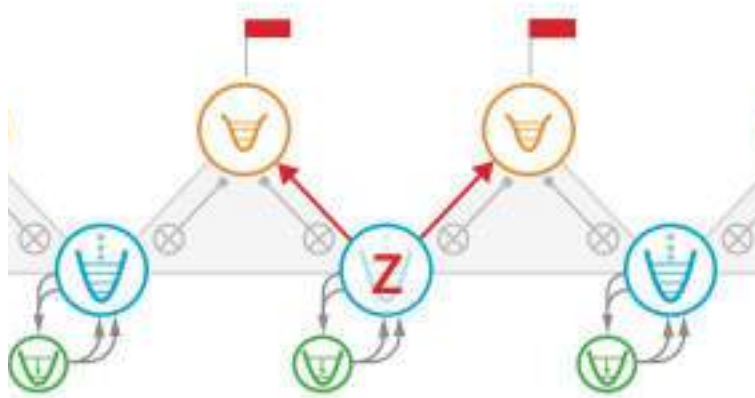


Guillaud & Mirrahimi, PRX, 2019

Putterman *et al.*, Nature, 2025

$X_i X_j$ (joint parity) $\longrightarrow$ photon-loss detection

Repetition cat code



Guillaud, Mirrahimi, PRX, 2019

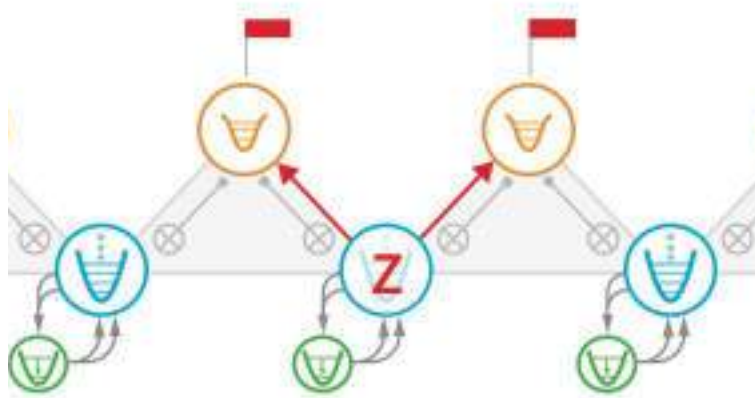
Putterman *et al.*, Nature, 2025

$$S_i = a_i^2 - \alpha^2 + 1 \quad (i = 1, 2, 3)$$

Distant vs photon loss

	$\begin{matrix} \nearrow \\ \nwarrow \end{matrix}$	$ c_{\alpha}^{-}\rangle c_{\alpha}^{+}\rangle c_{\alpha}^{+}\rangle$	$\longleftrightarrow$	$ c_{\alpha}^{-}\rangle c_{\alpha}^{-}\rangle c_{\alpha}^{+}\rangle$	$\begin{matrix} \nwarrow \\ \nearrow \end{matrix}$	
$ +X\rangle = c_{\alpha}^{+}\rangle c_{\alpha}^{+}\rangle c_{\alpha}^{+}\rangle$	$\longleftrightarrow$	$ c_{\alpha}^{+}\rangle c_{\alpha}^{-}\rangle c_{\alpha}^{+}\rangle$	$\begin{matrix} \times \\ \times \end{matrix}$	$ c_{\alpha}^{-}\rangle c_{\alpha}^{+}\rangle c_{\alpha}^{-}\rangle$	$\longleftrightarrow$	$ -X\rangle = c_{\alpha}^{-}\rangle c_{\alpha}^{-}\rangle c_{\alpha}^{-}\rangle$
	$\begin{matrix} \nwarrow \\ \nearrow \end{matrix}$	$ c_{\alpha}^{+}\rangle c_{\alpha}^{+}\rangle c_{\alpha}^{-}\rangle$	$\longleftrightarrow$	$ c_{\alpha}^{+}\rangle c_{\alpha}^{-}\rangle c_{\alpha}^{-}\rangle$	$\begin{matrix} \nwarrow \\ \nearrow \end{matrix}$	

Repetition cat code



Guillaud, Mirrahimi, PRX, 2019

Putterman *et al.*, Nature, 2025

$$S_i = a_i^2 - \alpha^2 + 1 \quad (i = 1, 2, 3)$$

$$\begin{aligned} | +Z \rangle = & | +\alpha \rangle | +\alpha \rangle | +\alpha \rangle + \\ & | +\alpha \rangle | -\alpha \rangle | -\alpha \rangle + \\ & | -\alpha \rangle | +\alpha \rangle | -\alpha \rangle + \\ & | -\alpha \rangle | -\alpha \rangle | +\alpha \rangle \end{aligned}$$

Distant supports



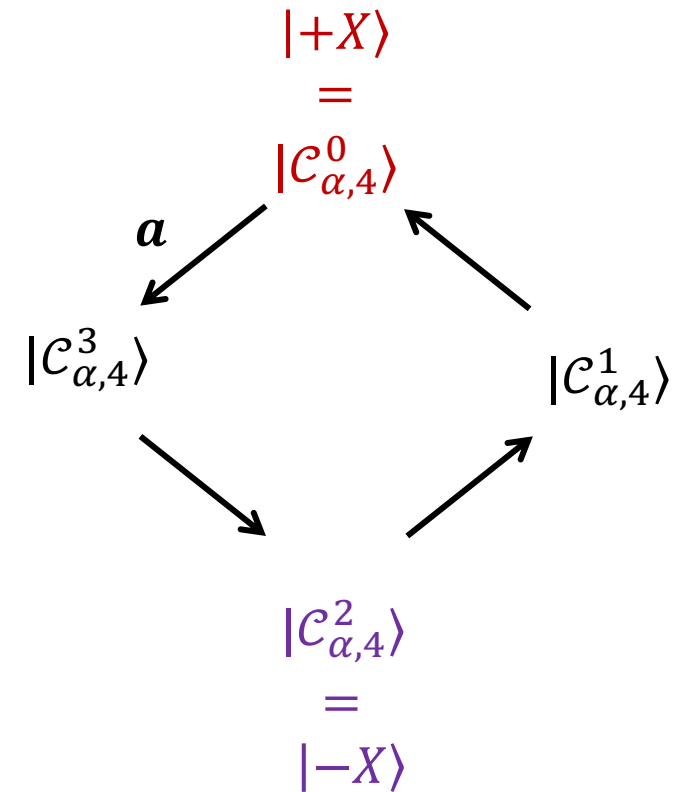
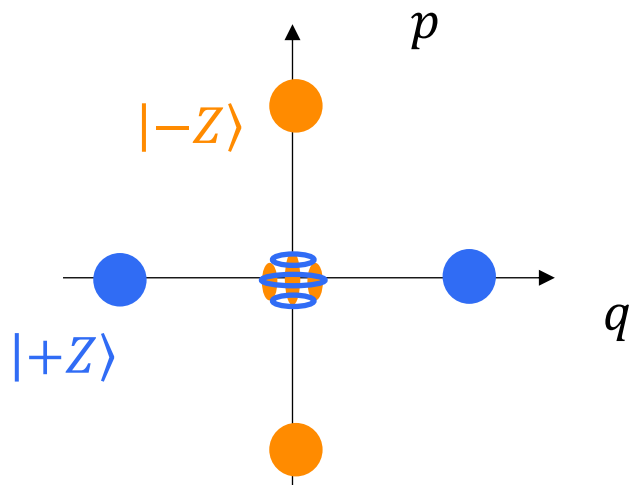
$$\begin{aligned} | -Z \rangle = & | +\alpha \rangle | +\alpha \rangle | -\alpha \rangle + \\ & | +\alpha \rangle | -\alpha \rangle | +\alpha \rangle + \\ & | -\alpha \rangle | +\alpha \rangle | +\alpha \rangle + \\ & | -\alpha \rangle | -\alpha \rangle | -\alpha \rangle \end{aligned}$$

4-legged cat qubits

Code : $\text{Span}\{|\mathcal{C}_\alpha^+\rangle, |\mathcal{C}_{i\alpha}^+\rangle\}$

Stabilizers: $S_{\alpha,4} = a^4 - \alpha^4 + 1$

$S_{\Pi,4} = e^{i\pi a^\dagger a} \longrightarrow$ Photon-loss detection

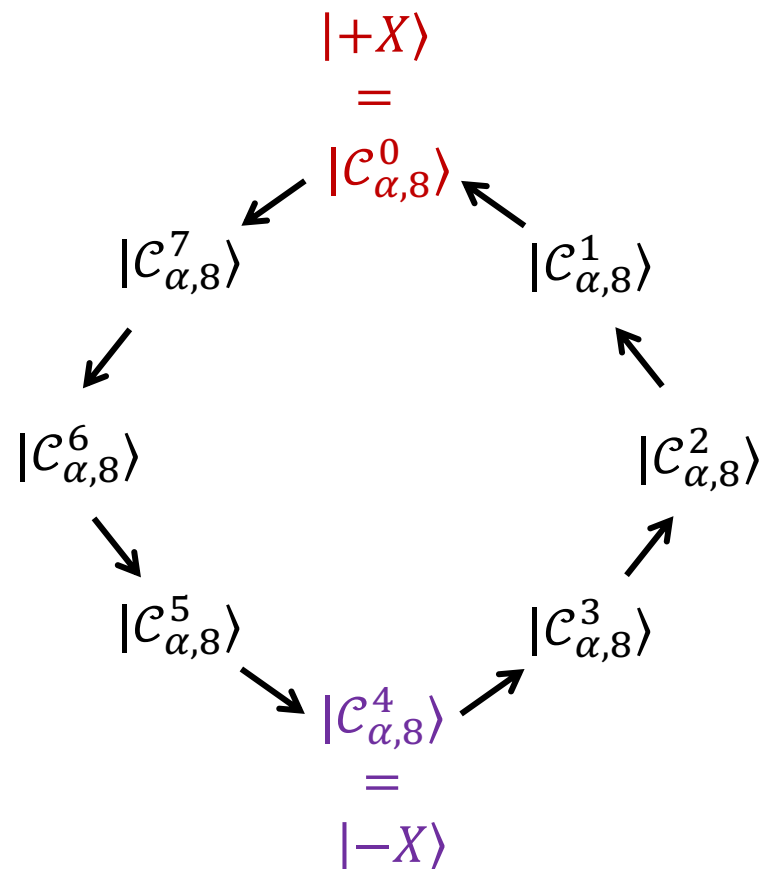
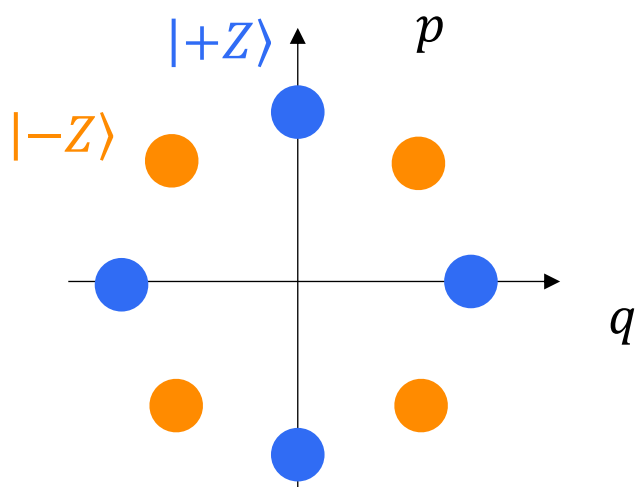


8-legged cat qubits

Code : $\text{Span}\{|c_{\alpha,4}^0\rangle, |c_{e^{i\pi/4}\alpha,4}^0\rangle\}$

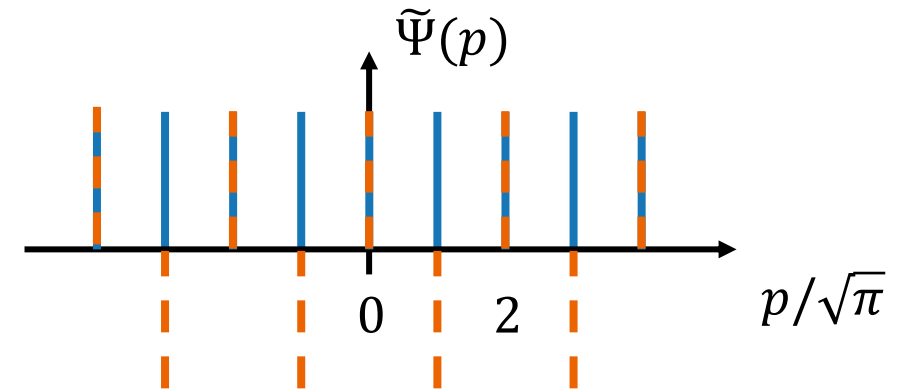
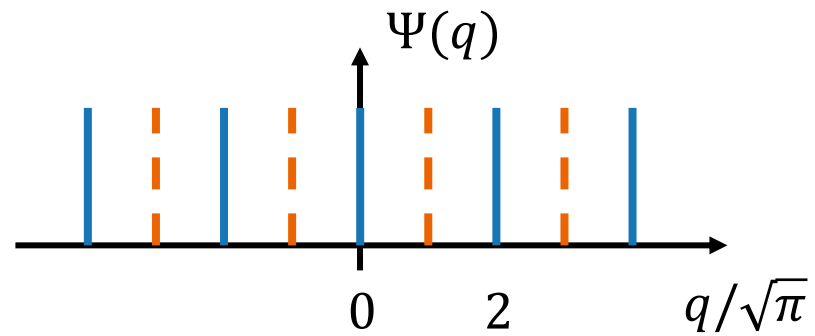
Stabilizers: $S_{\alpha,8} = a^8 - \alpha^8 + 1$

$S_{\Pi,8} = e^{\frac{i\pi}{2}a^\dagger a} \longrightarrow$ Photon-loss detection

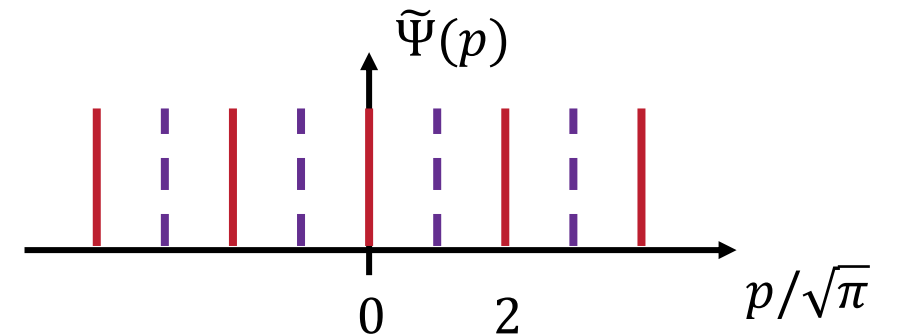
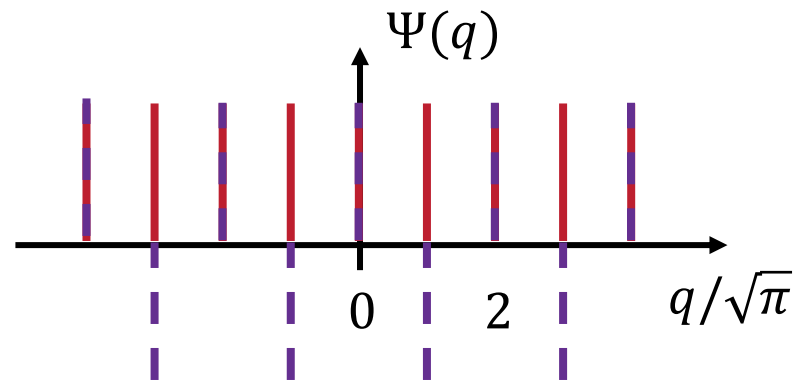


The GKP code

— $|+Z\rangle$
 - - $|-Z\rangle$



— $|+X\rangle$
 - - $|-X\rangle$



Stabilizers

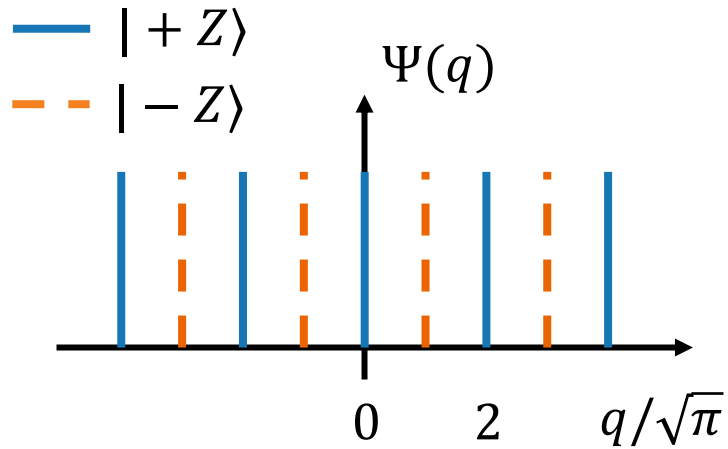
$$\begin{aligned} S_q &= e^{i2\sqrt{\pi}q} \\ S_p &= e^{-i2\sqrt{\pi}p} \end{aligned} \quad \text{or} \quad \begin{aligned} \tilde{S}_q &= q \bmod \sqrt{\pi} \\ \tilde{S}_p &= p \bmod \sqrt{\pi} \end{aligned}$$

Logical operators

$$\begin{aligned} Z &= e^{i\sqrt{\pi}q} \\ X &= e^{-i\sqrt{\pi}p} \\ Y &= e^{i\sqrt{\pi}(q-p)} \end{aligned}$$

The finite-energy GKP code

Infinite-energy code



$$S_q = e^{i2\sqrt{\pi}q}$$

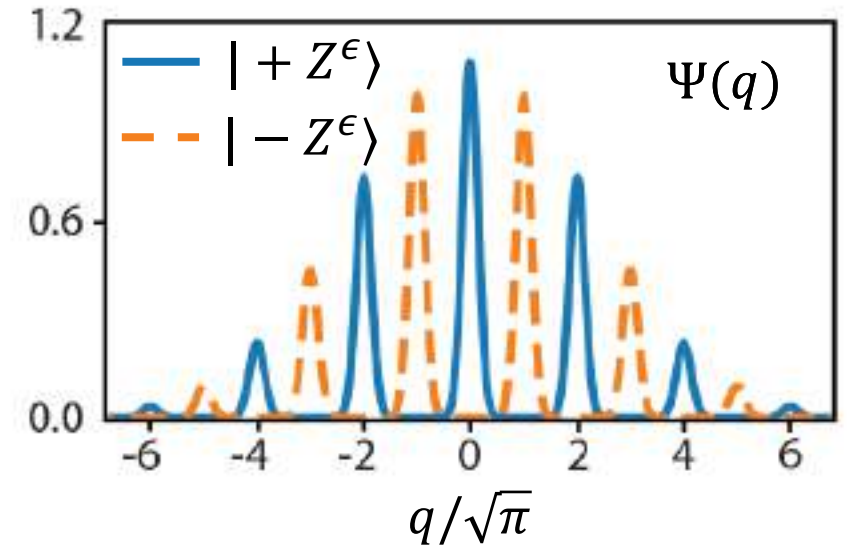
$$S_p = e^{-i2\sqrt{\pi}p}$$

$$O = e^{-\frac{\epsilon}{4\sqrt{\pi}}(q^2 + p^2)}$$

$$|\Psi_\epsilon\rangle = O|\Psi\rangle$$

$$S_\epsilon = OSO^{-1}$$

Finite-energy code



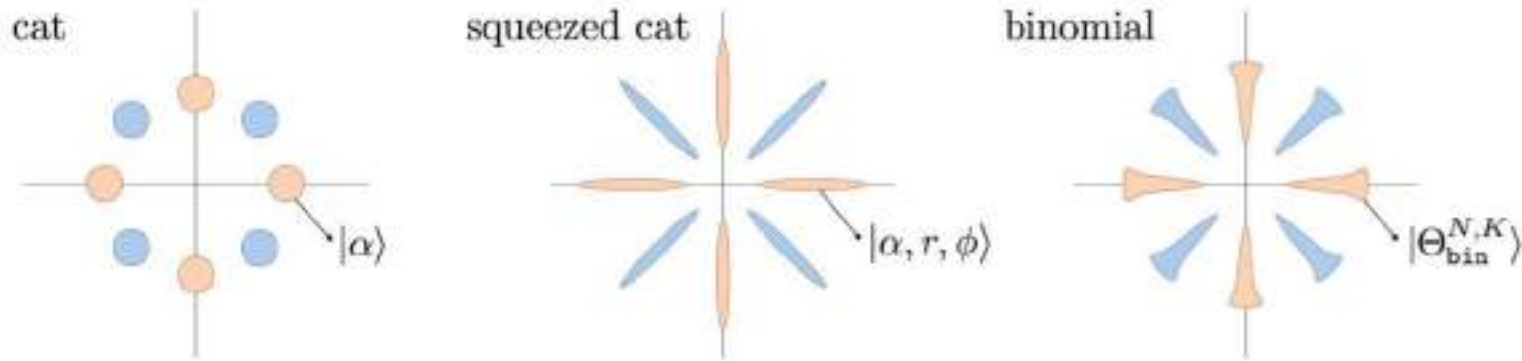
$$S_q^\epsilon = e^{i2\sqrt{\pi}q - \epsilon p}$$

$$S_p^\epsilon = e^{-i2\sqrt{\pi}p - \epsilon q}$$

Advanced bosonic qubits

Rotation-symmetric codes

Grimsmo, Combes, Baragiola., PRX 2020

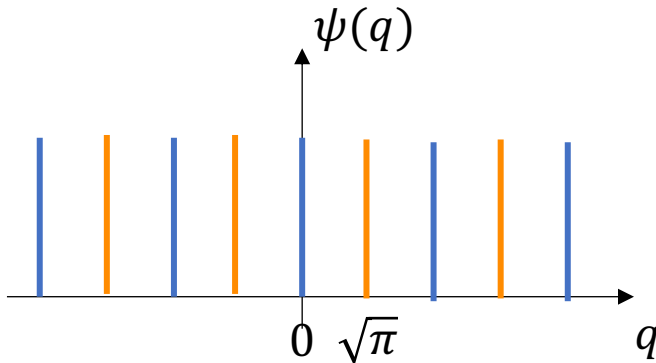


$$S_n = U(a^n - \alpha^n + 1)U^\dagger$$

$$S_{\Pi,n} = e^{i\frac{4\pi}{n}a^\dagger a}$$

Translation-symmetric codes

Gottesman, Kitaev, Preskill, PRA 2001



$$S_q = e^{i2\sqrt{\pi}q}$$

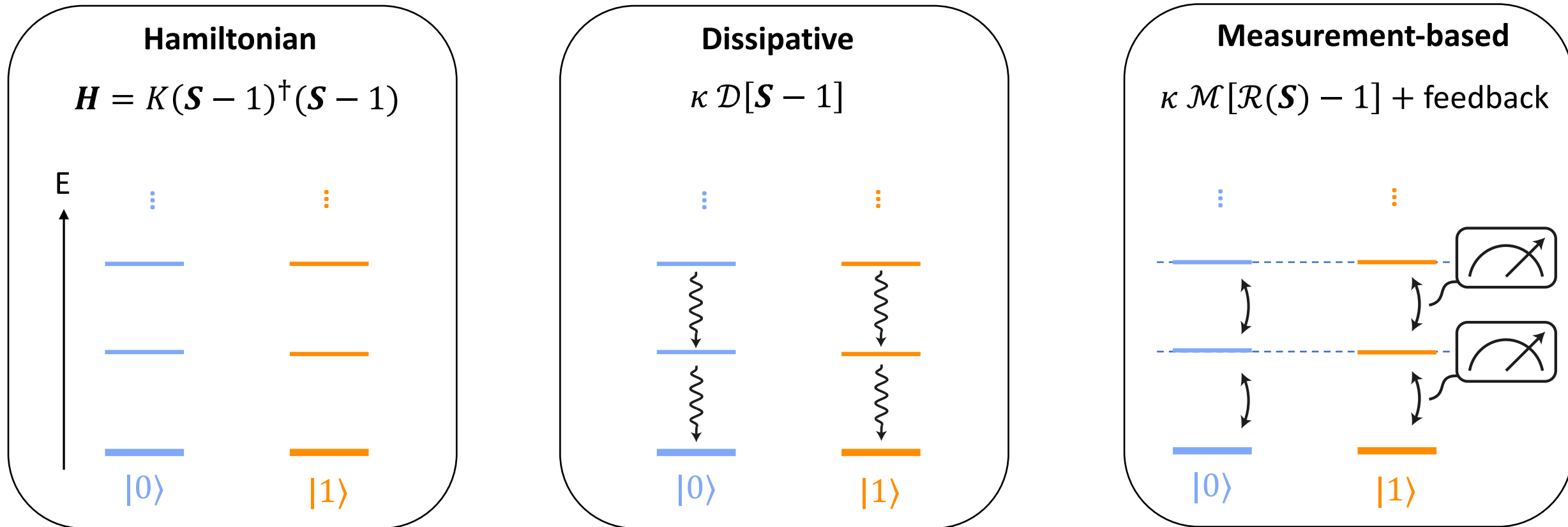
$$S_p = e^{i2\sqrt{\pi}p}$$

High-order stabilizers

Outline

1. Bosonic codes: structure
- 2. Bosonic codes: dissipative stabilization**
3. First experimental steps

Stabilization flavors



+ colored bath
Putterman, PRL, 2022

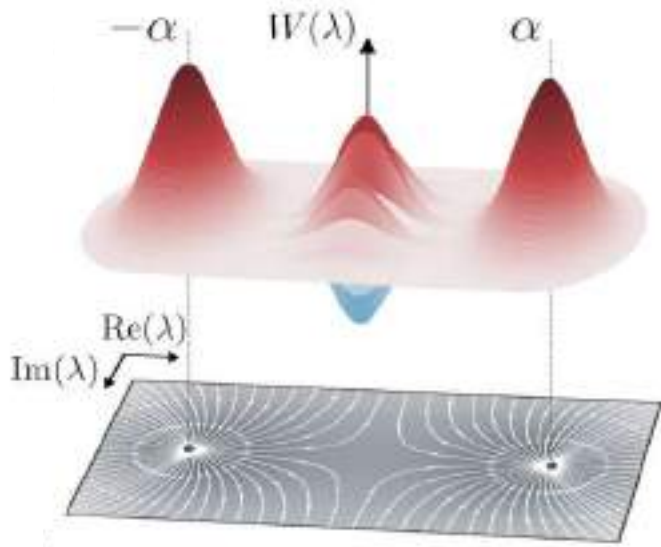
If feedback $\propto \Im(S)$
Chia, Wiseman, PRA 2011

Two-legged cat qubits

Code : $\text{Span}\{|+\alpha\rangle, |-\alpha\rangle\}$

Stabilizer: $S = a^2 - \alpha^2 + 1$

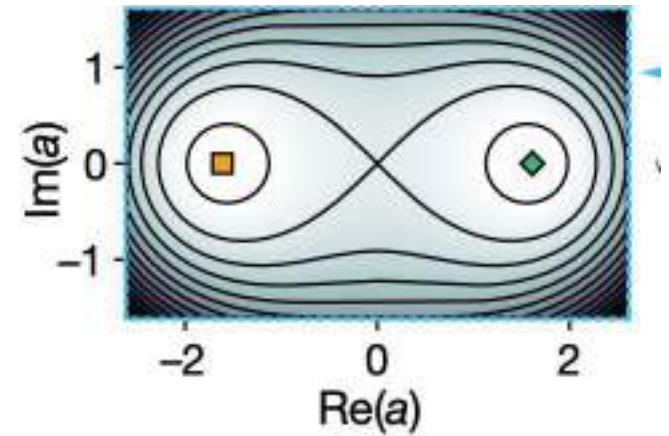
Dissipative



$$\kappa_2 \mathcal{D}[a^2 - \alpha^2]$$

Réglade *et al.*, Nature 2024

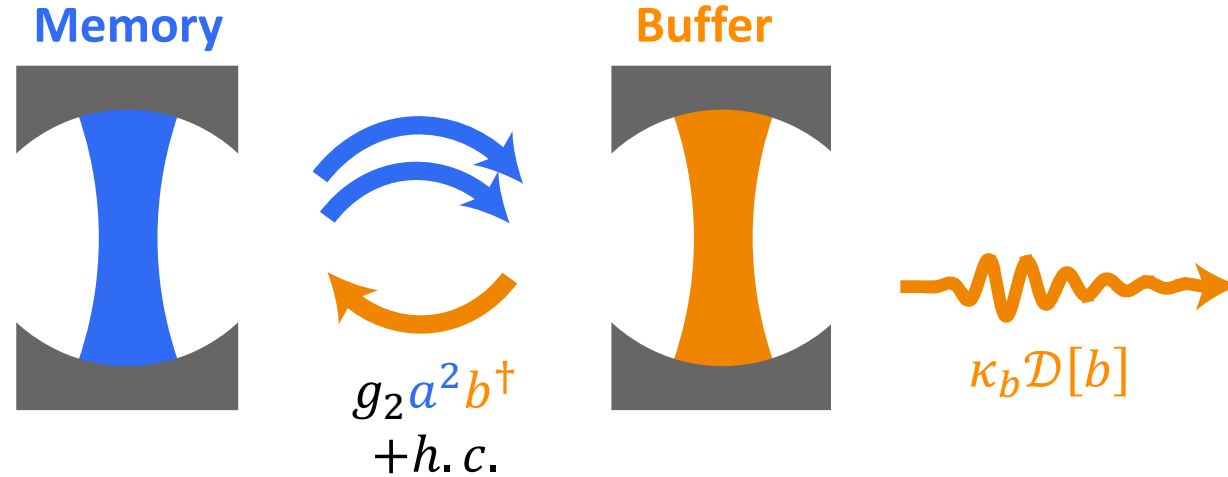
Hamiltonian



$$H = -K (a^{\dagger 2} - \alpha^2)(a^2 - \alpha^2)$$

Grimm *et al.*, Nature 2020

Dissipative stabilization



$$H_{int} = g_2 a^2 b^\dagger + h.c.$$

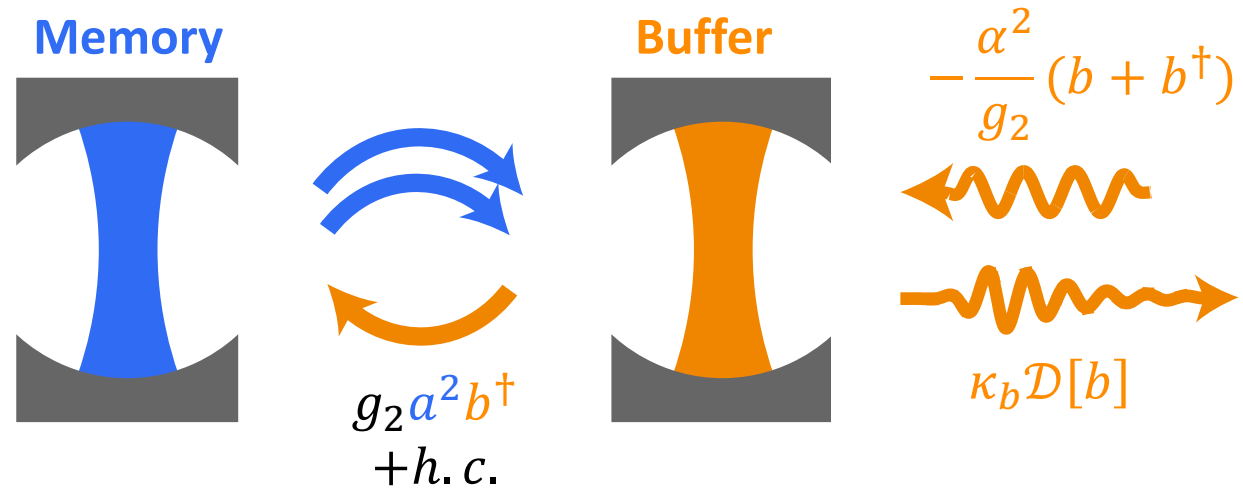
$$g_2 \ll \kappa_b$$

→
Adiabatic
elimination

$$\kappa_2 \mathcal{D}[a^2]$$

$$\kappa_2 = \frac{4g_2^2}{\kappa_b}$$

Dissipative stabilization



$$H_{int} = g_2 (a^2 - \alpha^2) b^\dagger + h.c.$$

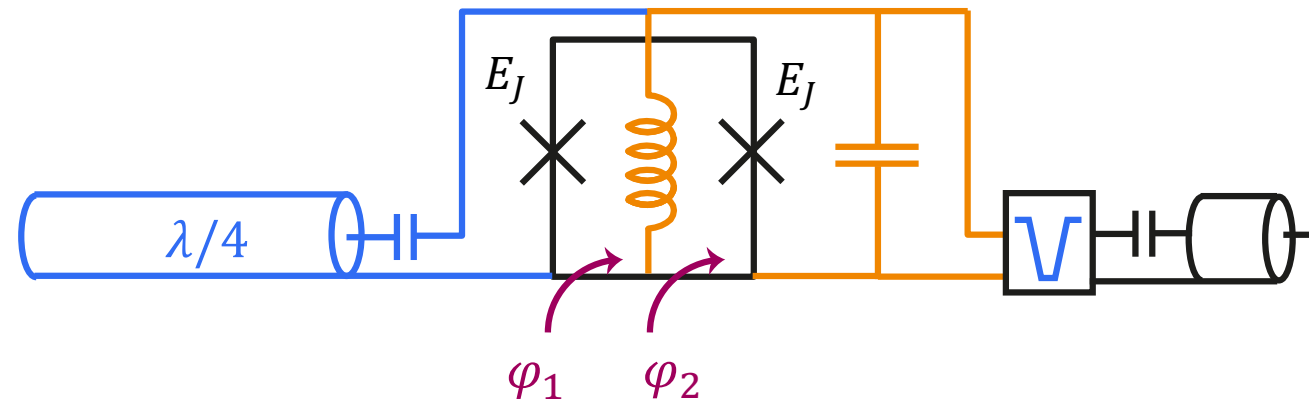
$$g_2 \ll \kappa_b$$

→
Adiabatic
elimination

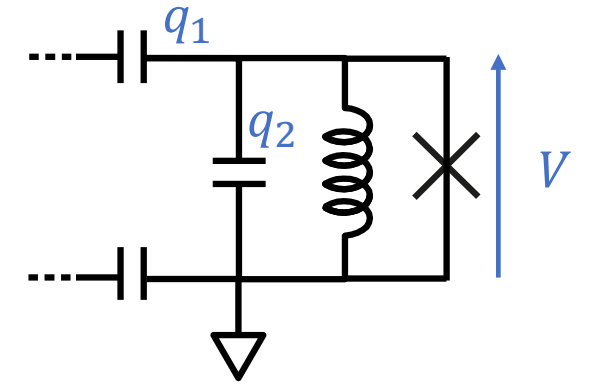
$$\kappa_2 \mathcal{D}[a^2 - \alpha^2]$$

$$\kappa_2 = \frac{4g_2^2}{\kappa_b}$$

2-photon exchange



circuitQED cheat sheet



$$\Phi = \varphi_0 \varphi = \int_{-\infty}^t V(t') dt'$$

$$Q = q_1 + q_2$$

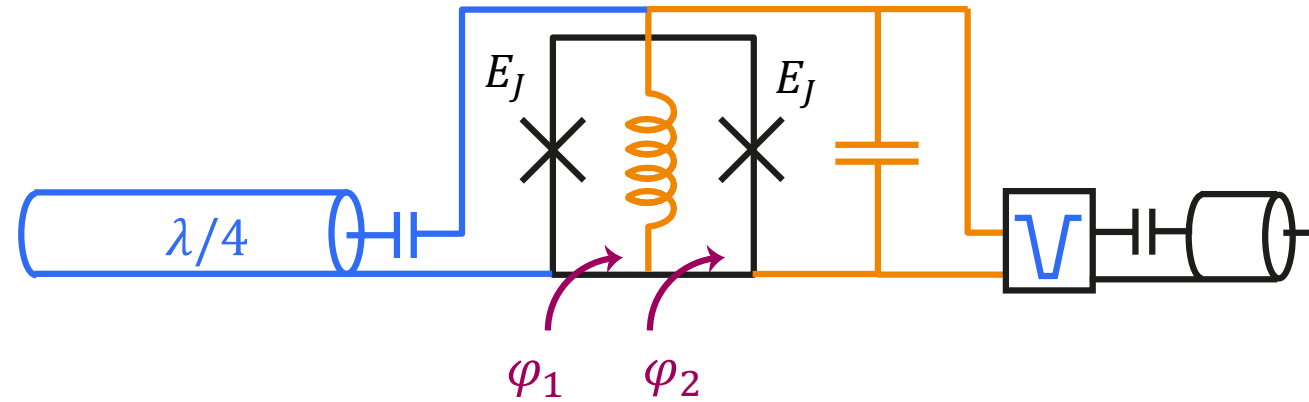
$$[\Phi, Q] = i\hbar$$

$$U_C = \frac{1}{2C} q^2$$

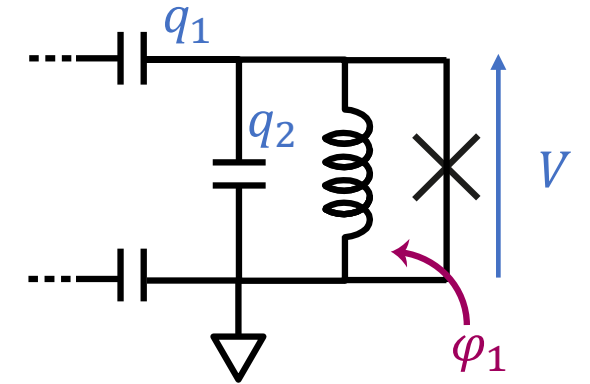
$$U_L = \frac{1}{2L} \Phi^2$$

$$U_J = -E_J \cos(\varphi)$$

2-photon exchange



circuitQED cheat sheet



$$\Phi = \varphi_0 \varphi = \int_{-\infty}^t V(t') dt'$$

$$Q = q_1 + q_2$$

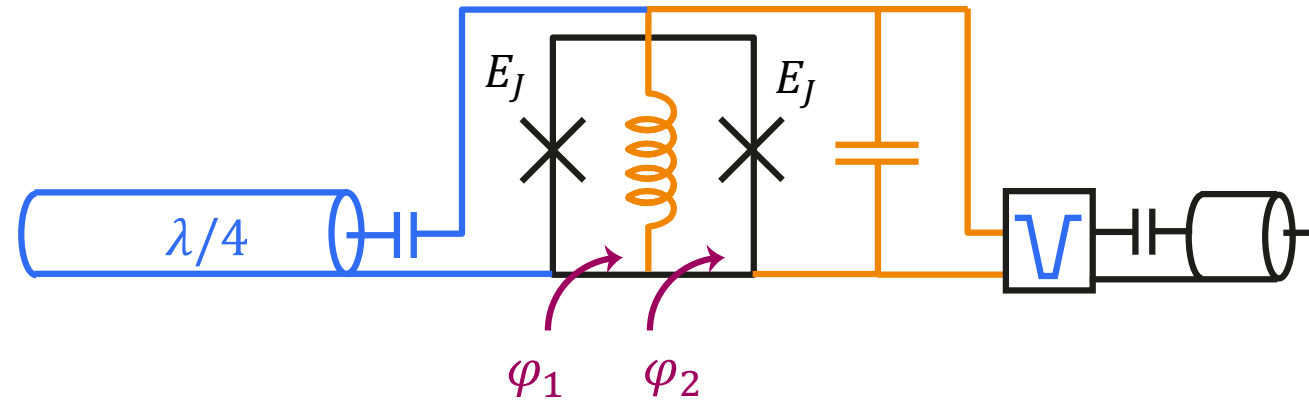
$$[\Phi, Q] = i\hbar$$

$$U_C = \frac{1}{2C} q^2$$

$$U_L = \frac{1}{2L} \Phi^2$$

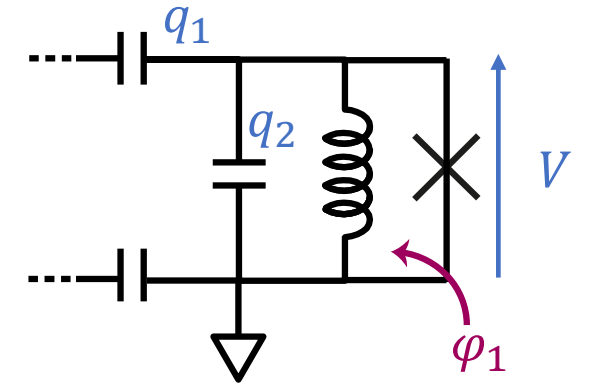
$$U_J = -E_J \cos(\varphi + \varphi_1)$$

2-photon exchange



$$H = H_{quad} - E_J \cos(\varphi_{J_1} - \varphi_1) - E_J \cos(\varphi_{J_2} + \varphi_2)$$

circuitQED cheat sheet



$$\Phi = \varphi_0 \varphi = \int_{-\infty}^t V(t') dt'$$

$$Q = q_1 + q_2$$

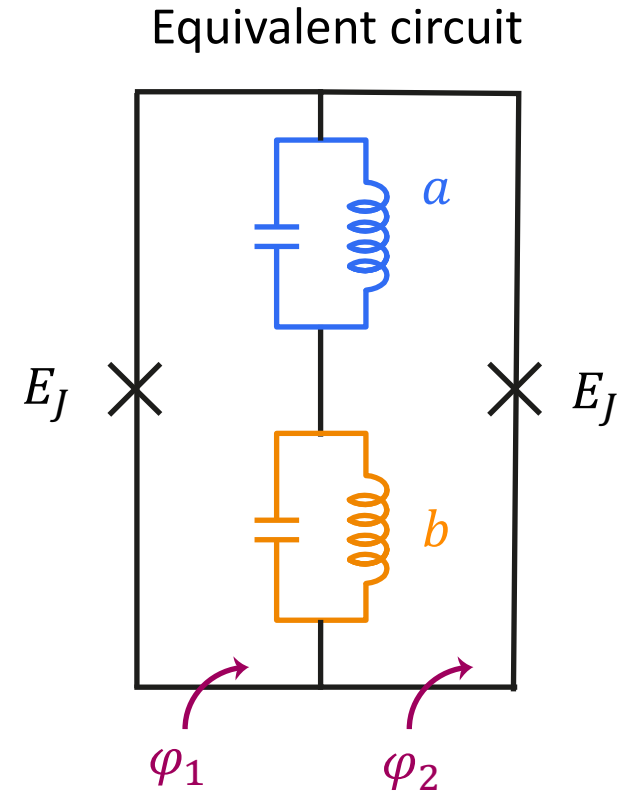
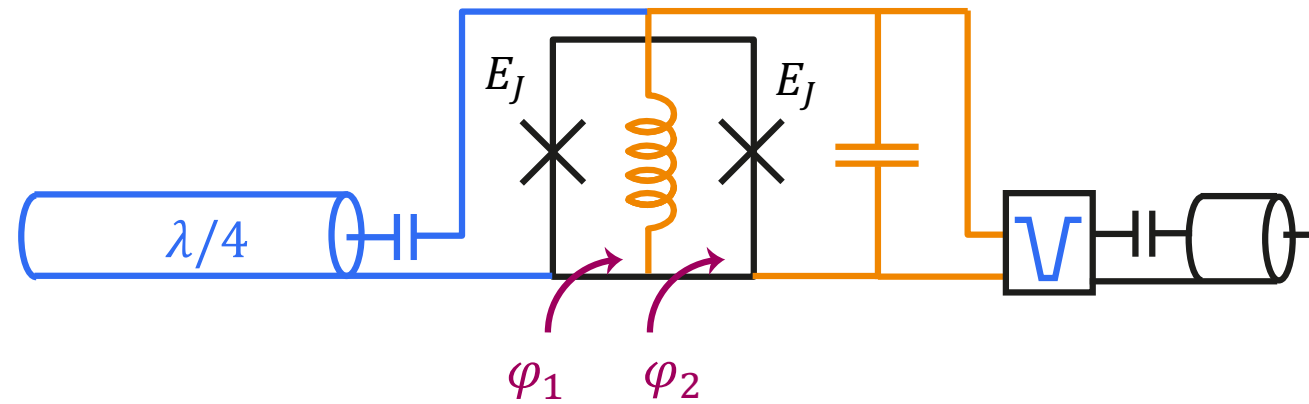
$$[\Phi, Q] = i\hbar$$

$$U_C = \frac{1}{2C} q^2$$

$$U_L = \frac{1}{2L} \Phi^2$$

$$U_J = -E_J \cos(\varphi + \varphi_1)$$

2-photon exchange



$$H = \omega_a a^\dagger a + \omega_b b^\dagger b - E_J \cos(\varphi_a + \varphi_b - \varphi_1) - E_J \cos(\varphi_a + \varphi_b + \varphi_2)$$

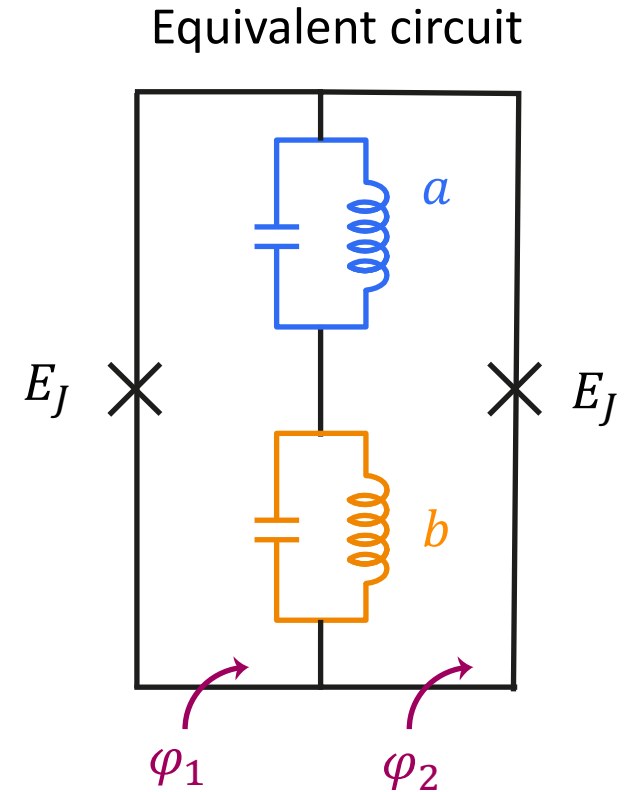
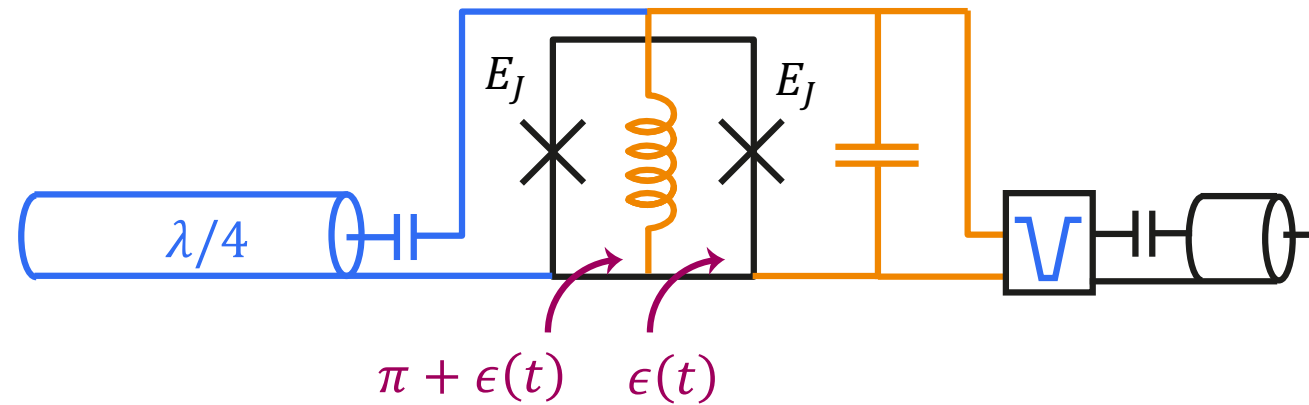
$$\varphi_{ZPF,a} = \sqrt{\pi Z_a / R_q}$$

$$\varphi_a = \varphi_{ZPF,a}(a + a^\dagger)$$

$$\varphi_{ZPF,b} = \sqrt{\pi Z_b / R_q}$$

$$\varphi_b = \varphi_{ZPF,b}(b + b^\dagger)$$

2-photon exchange



$$H = \omega_a a^\dagger a + \omega_b b^\dagger b - E_J \cos(\varphi_a + \varphi_b - \pi - \epsilon(t)) - E_J \cos(\varphi_a + \varphi_b + \epsilon(t))$$

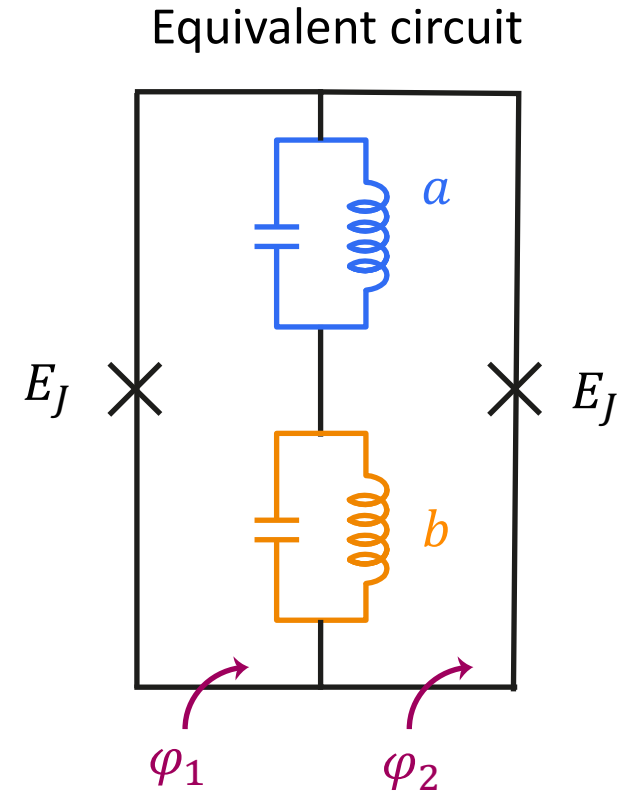
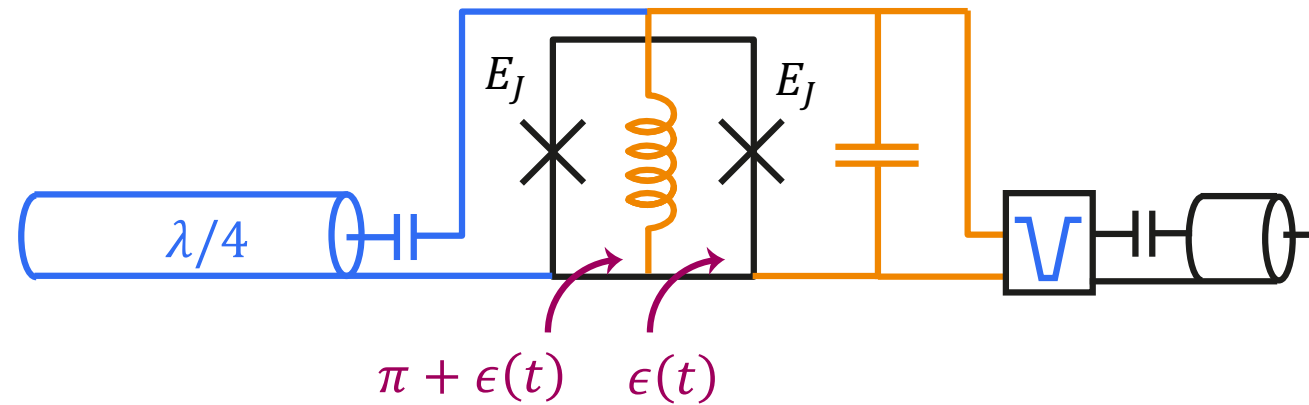
$$\varphi_{ZPF,a} = \sqrt{\pi Z_a / R_q}$$

$$\varphi_a = \varphi_{ZPF,a}(a + a^\dagger)$$

$$\varphi_{ZPF,b} = \sqrt{\pi Z_b / R_q}$$

$$\varphi_b = \varphi_{ZPF,b}(b + b^\dagger)$$

2-photon exchange



$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + 2E_J \sin(\epsilon(t)) \sin(\varphi_a + \varphi_b)$$

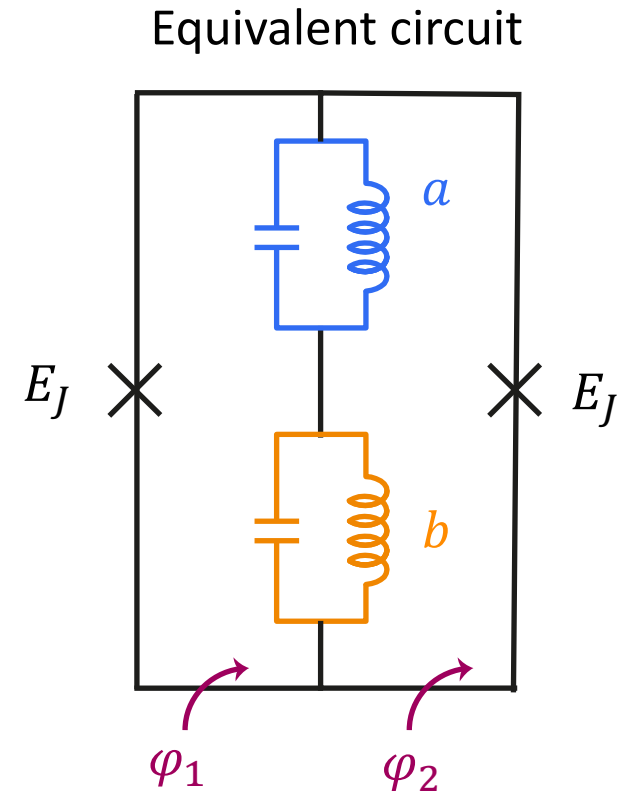
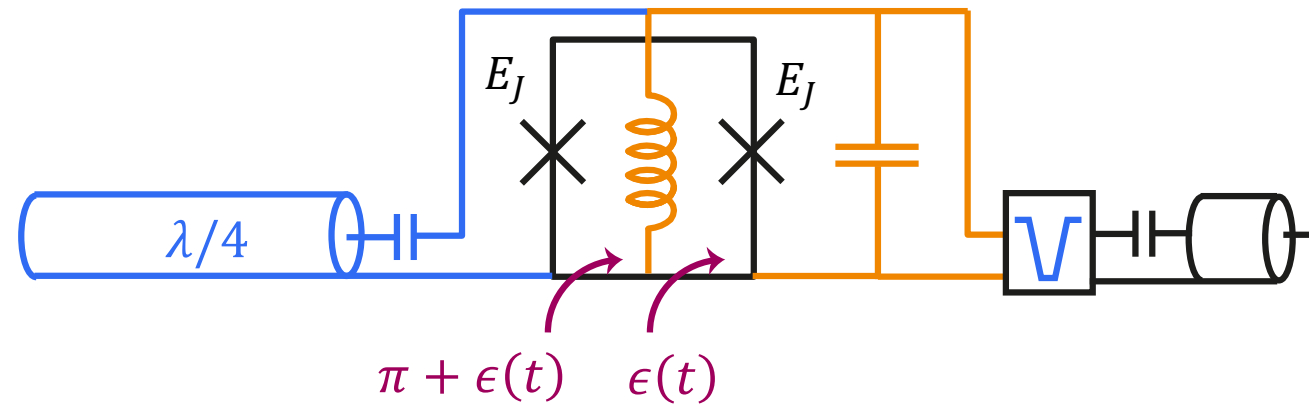
$$\varphi_{ZPF,a} = \sqrt{\pi Z_a / R_q}$$

$$\varphi_a = \varphi_{ZPF,a} (a + a^\dagger)$$

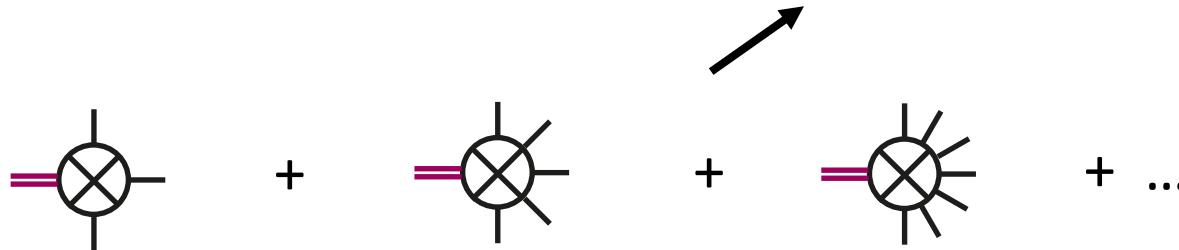
$$\varphi_{ZPF,b} = \sqrt{\pi Z_b / R_q}$$

$$\varphi_b = \varphi_{ZPF,b} (b + b^\dagger)$$

2-photon exchange



$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + 2E_J \sin(\epsilon(t)) \sin(\varphi_a + \varphi_b)$$

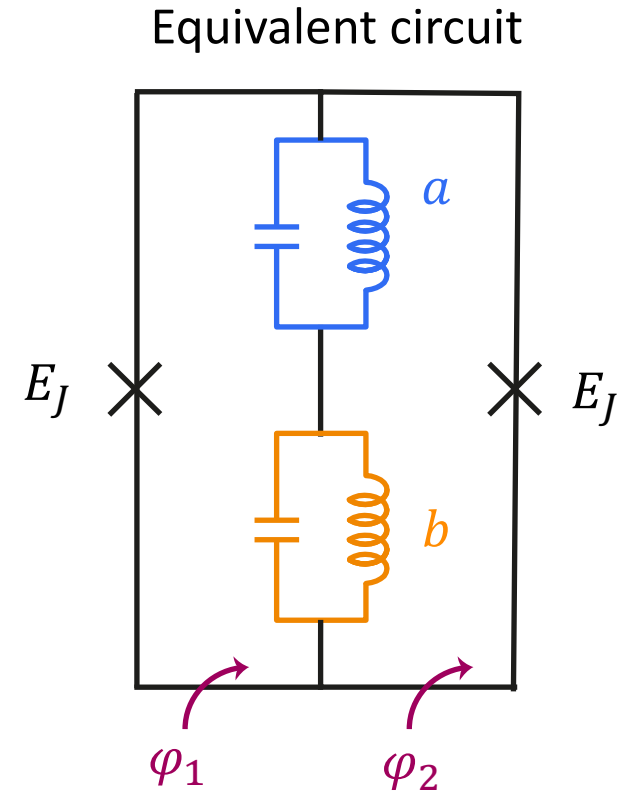


$$\varphi_a = \varphi_{ZPF,a}(a + a^\dagger)$$

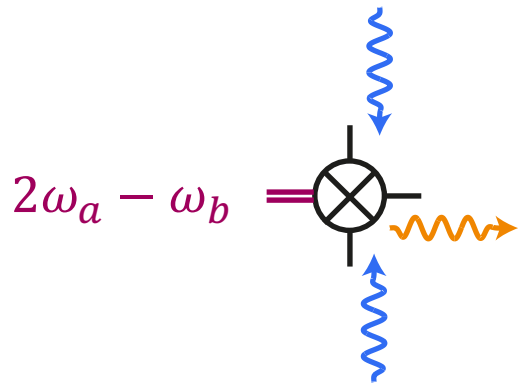
$$\varphi_b = \varphi_{ZPF,b}(b + b^\dagger)$$

2-photon exchange

Kerr-free
Operated at sweet-spot



$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + 2E_J \sin(\epsilon(t)) \sin(\varphi_a + \varphi_b)$$



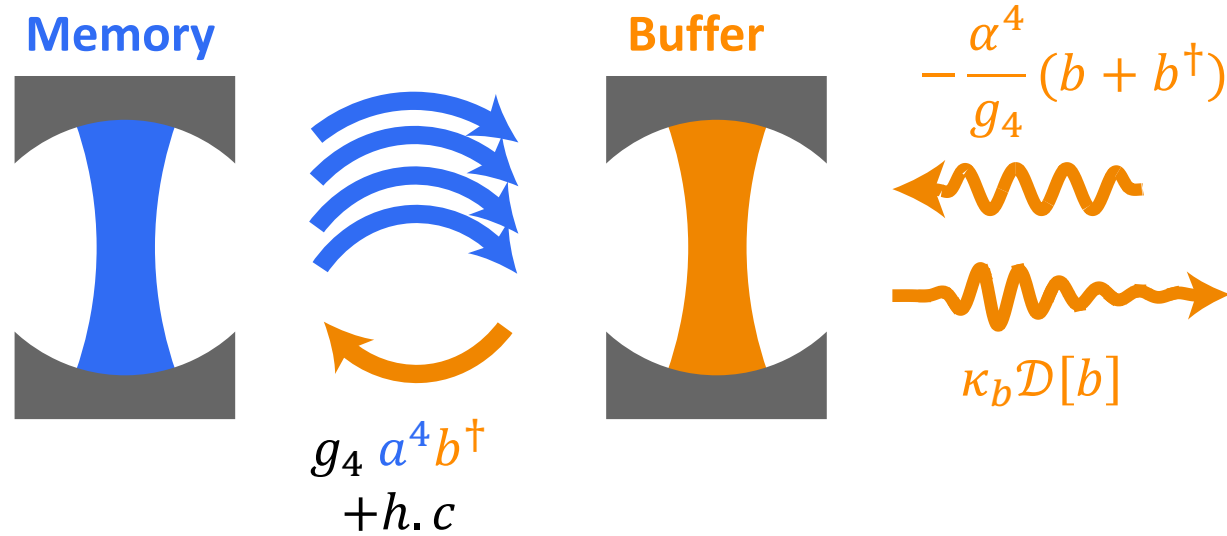
Rotate +
RWA

$$\varphi_{ZPF,a}^2 a^2 \varphi_{ZPF,b} b^\dagger + h.c. \\ + O(\varphi_{ZPF}^5)$$

$$\varphi_a = \varphi_{ZPF,a}(a + a^\dagger)$$

$$\varphi_b = \varphi_{ZPF,b}(b + b^\dagger)$$

Dissipative stabilization of 4-legged cats



$$H_{int} = g_4 (a^4 - \alpha^4) b^\dagger + h.c.$$

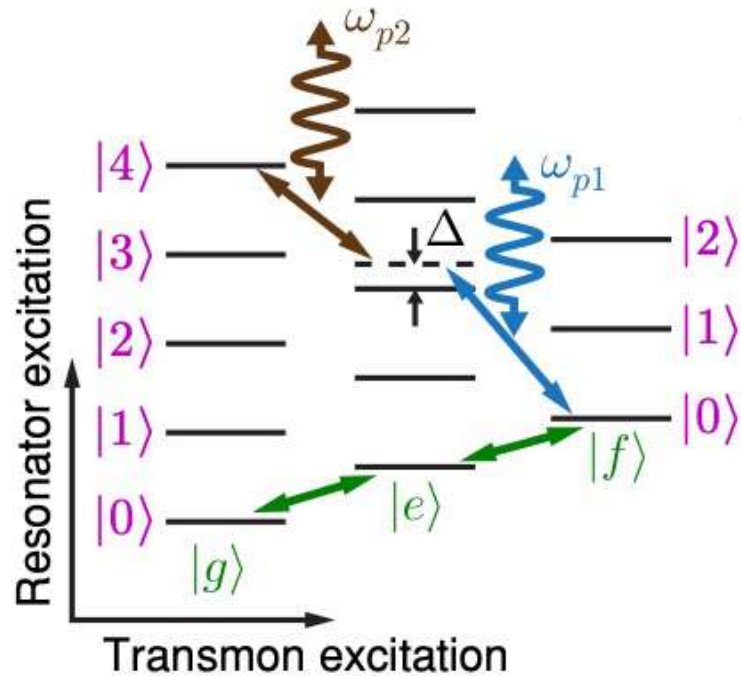
$$g_4 \ll \kappa_b$$

→
Adiabatic
elimination

$$\kappa_4 \mathcal{D}[a^4 - \alpha^4]$$

$$\kappa_4 = \frac{4g_4^2}{\kappa_b}$$

From low-order to high-order interaction



$$a^2|e\rangle\langle g|e^{i\Delta t} + h.c.$$

$$a^2|f\rangle\langle e|e^{-i\Delta t} + h.c.$$

Raman cascade



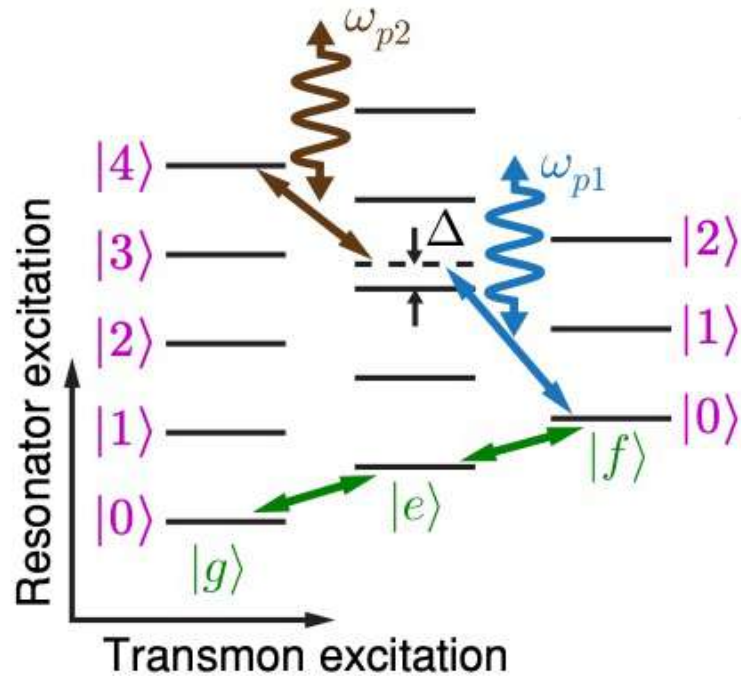
$$a^4|f\rangle\langle g| + h.c.$$

Adiabatic
elimination



$$\kappa_4 \mathcal{D}[a^4]$$

From low-order to high-order interaction



$$a^2|e\rangle\langle g|e^{i\Delta t} + h.c.$$

$$a^2|f\rangle\langle e|e^{-i\Delta t} + h.c.$$

Raman cascade



$$a^4|f\rangle\langle g| + h.c.$$

Adiabatic
elimination



$$\kappa_4 \mathcal{D}[a^4]$$

Mundhada *et al.*, PRA 2019

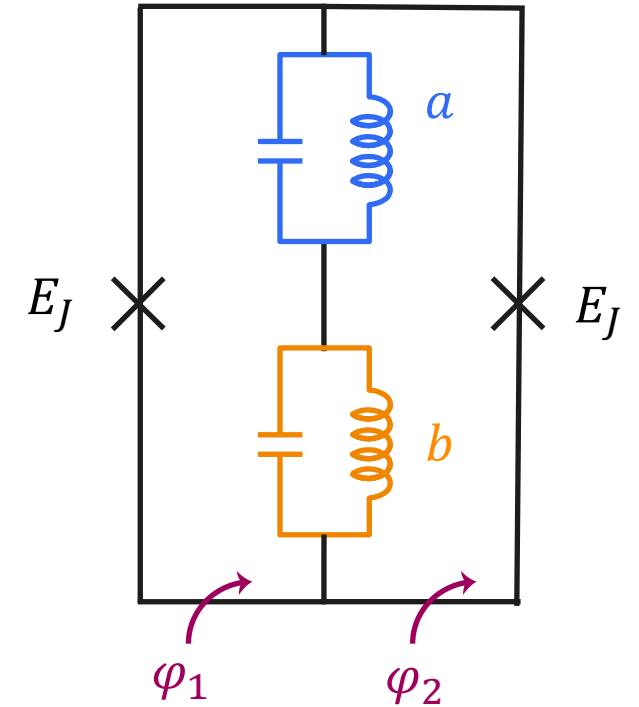
Finite Δ : residual $\kappa_2 \mathcal{D}[a^2]$

Genuine high-order interactions

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + 2E_J \sin(\epsilon(t)) \sin(\varphi_{ZPF,a}(a + a^\dagger) + \varphi_{ZPF,b}(b + b^\dagger))$$



Equivalent circuit

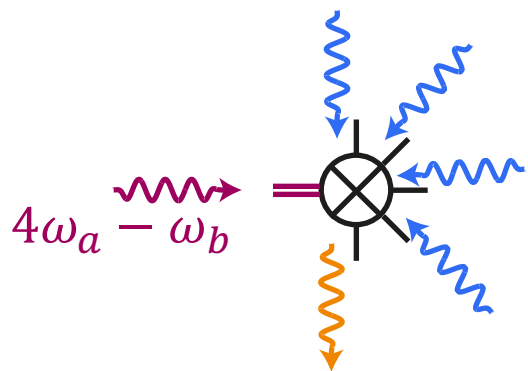


$$\varphi_{ZPF,a} = \sqrt{\pi Z_a / R_q}$$

$$\varphi_{ZPF,b} = \sqrt{\pi Z_b / R_q}$$

Genuine high-order interactions

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + 2E_J \sin(\epsilon(t)) \sin(\varphi_{ZPF,a}(a + a^\dagger) + \varphi_{ZPF,b}(b + b^\dagger))$$



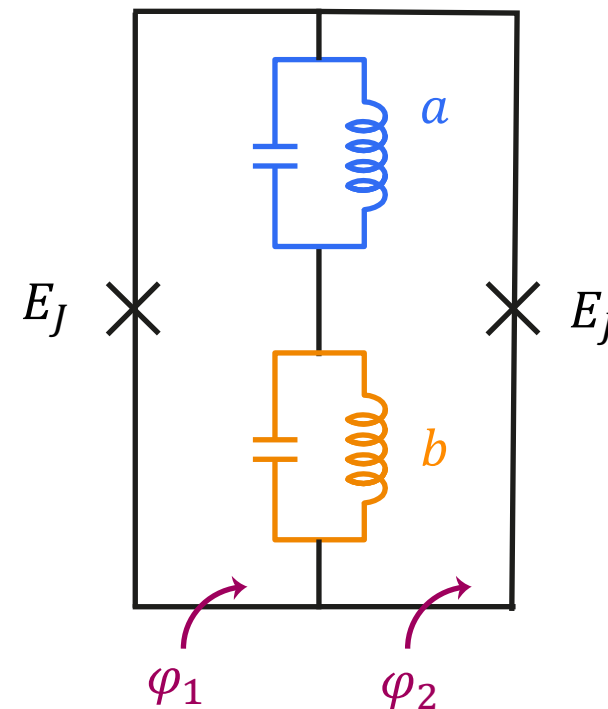
$$\varphi_{ZPF,a}^4 a^4 \varphi_{ZPF,b} b^\dagger + h.c.$$

$$\downarrow$$

$$\kappa_4 \mathcal{D}[a^4]$$

$$\left. \begin{array}{l} \varphi_{ZPF,a} \\ \varphi_{ZPF,b} \end{array} \right\} \sim 1$$

Equivalent circuit



$$\varphi_{ZPF,a} = \sqrt{\pi Z_a / R_q}$$

$$\varphi_{ZPF,b} = \sqrt{\pi Z_b / R_q}$$

Dissipative stabilization of GKP qubits

$$\mathcal{D}[\mathbf{S}_q^\epsilon - \mathbf{I}] \simeq \mathcal{D}[e^{i2\sqrt{\pi}\mathbf{q}}(\mathbf{I} - \epsilon\mathbf{p}) - \mathbf{I}]$$

$$\mathcal{D}[\mathbf{S}_p^\epsilon - \mathbf{I}] \simeq \mathcal{D}[e^{-i2\sqrt{\pi}\mathbf{p}}(\mathbf{I} - \epsilon\mathbf{q}) - \mathbf{I}]$$

Dissipative stabilization of GKP qubits

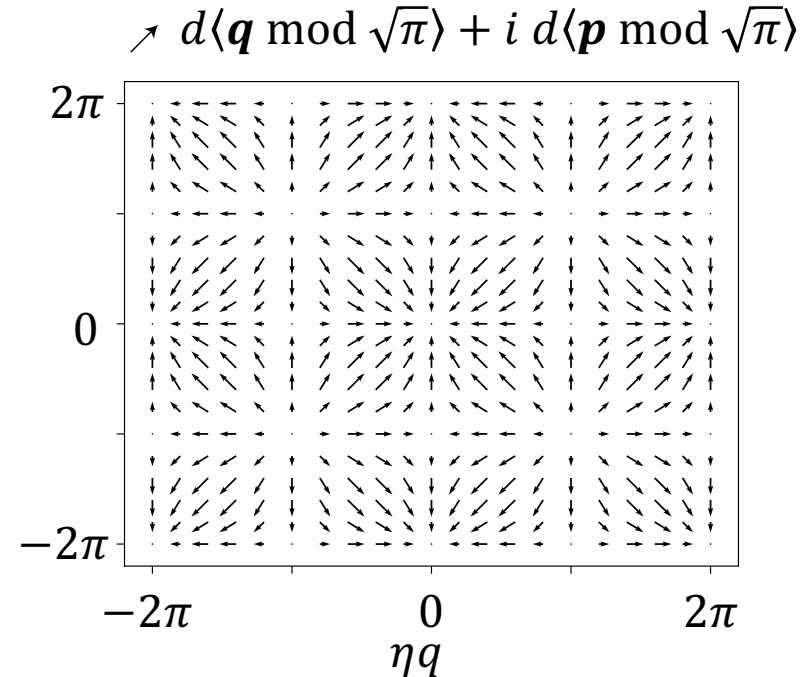
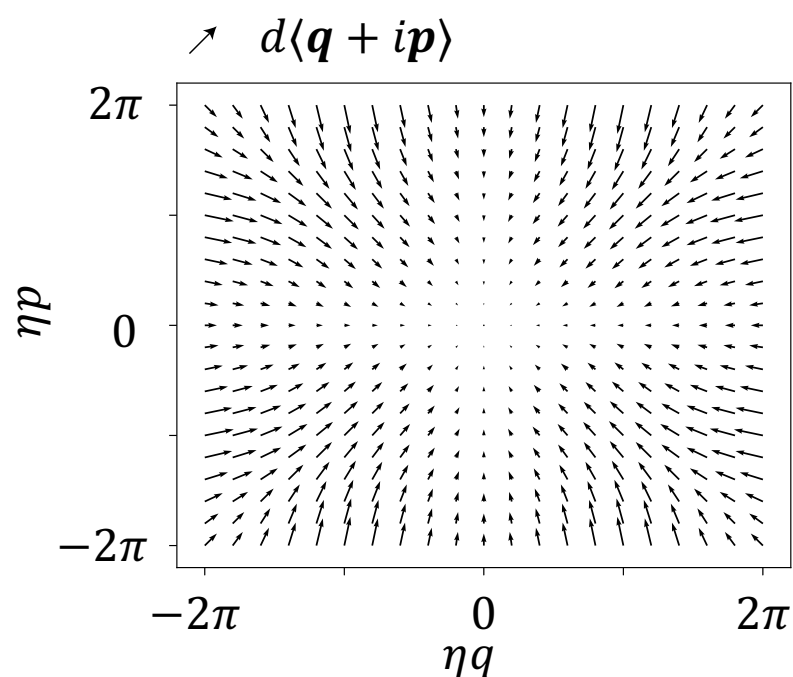
4-dissipator dynamics

$$\mathcal{D}[S_q^\epsilon - I] \simeq \mathcal{D}[e^{i2\sqrt{\pi}q}(I - \epsilon p) - I]$$

$$\mathcal{D}[S_p^\epsilon - I] \simeq \mathcal{D}[e^{-i2\sqrt{\pi}p}(I - \epsilon q) - I]$$

$$\mathcal{D}[S_{-q}^\epsilon - I] \simeq \mathcal{D}[e^{-i2\sqrt{\pi}q}(I + \epsilon p) - I]$$

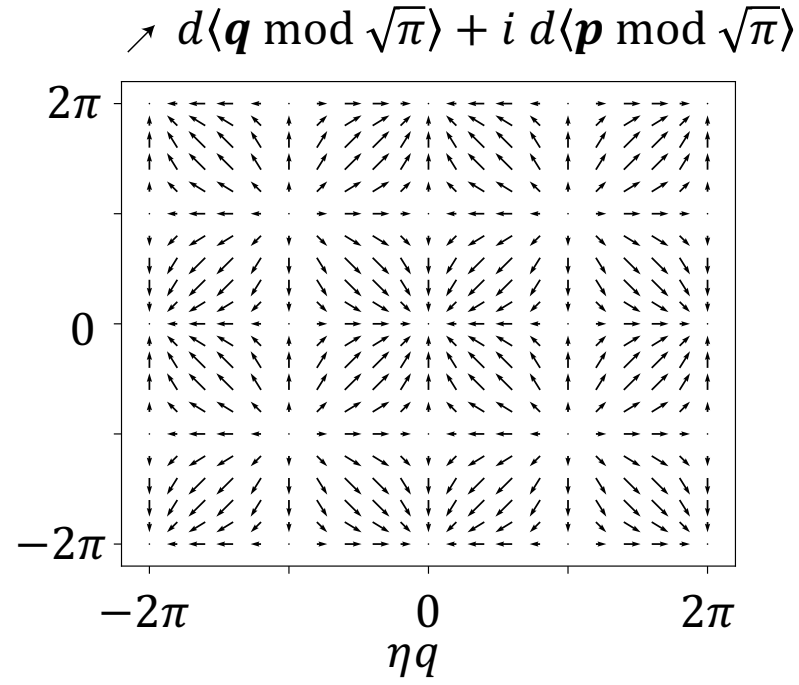
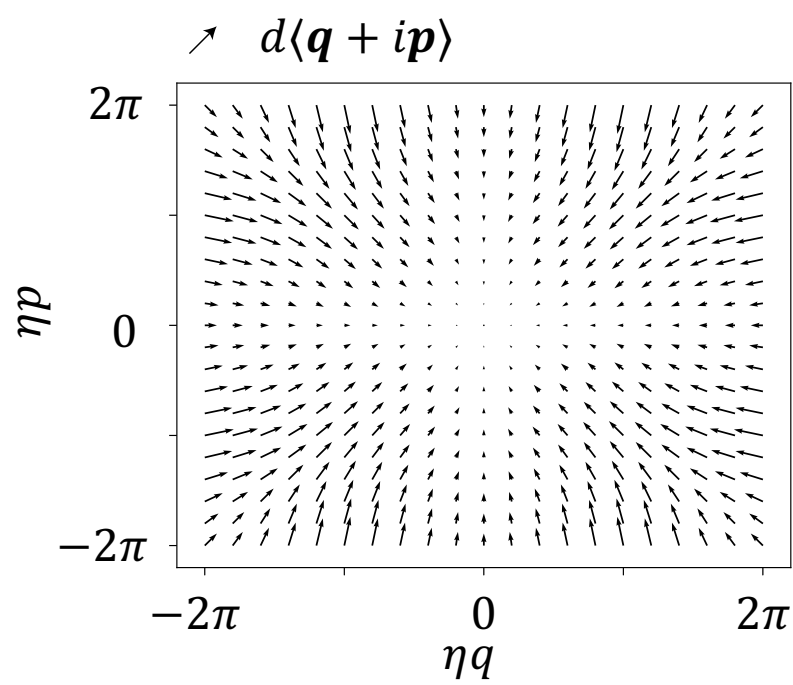
$$\mathcal{D}[S_{-p}^\epsilon - I] \simeq \mathcal{D}[e^{i2\sqrt{\pi}p}(I + \epsilon q) - I]$$



Dissipative stabilization of GKP qubits

4-dissipator dynamics

$$\left. \begin{aligned} \mathcal{D}[S_q^\epsilon - I] &\simeq \mathcal{D}[e^{i2\sqrt{\pi}q}(I - \epsilon p) - I] \\ \mathcal{D}[S_p^\epsilon - I] &\simeq \mathcal{D}[e^{-i2\sqrt{\pi}p}(I - \epsilon q) - I] \\ \mathcal{D}[S_{-q}^\epsilon - I] &\simeq \mathcal{D}[e^{-i2\sqrt{\pi}q}(I + \epsilon p) - I] \\ \mathcal{D}[S_{-p}^\epsilon - I] &\simeq \mathcal{D}[e^{i2\sqrt{\pi}p}(I + \epsilon q) - I] \end{aligned} \right\} \begin{array}{l} \text{from modular interactions} \\ g \cos(2\sqrt{\pi}q_a + \theta) \begin{array}{l} q_b \\ p_b \end{array} \end{array}$$

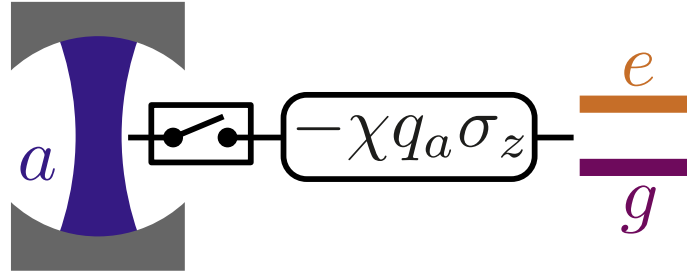


Modular interactions from low-order interactions

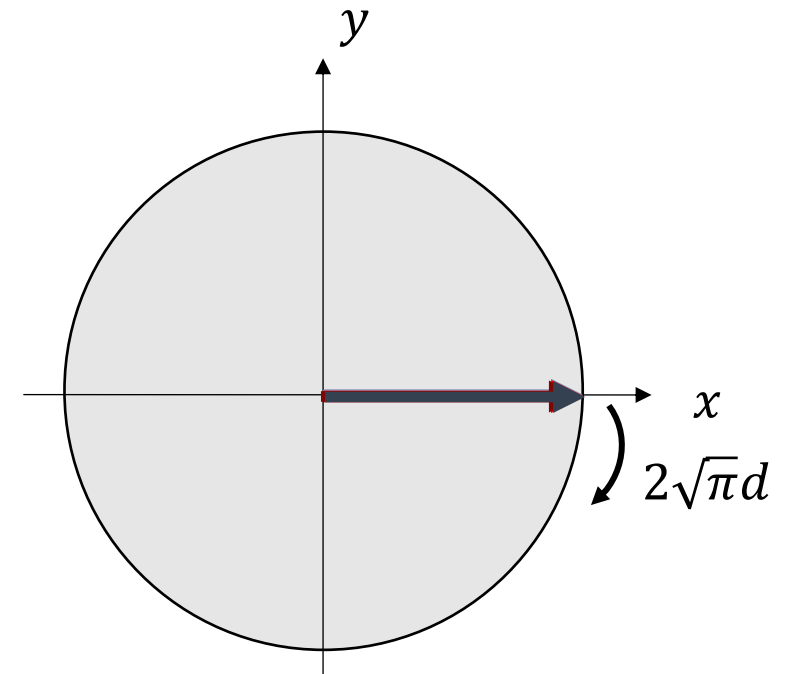
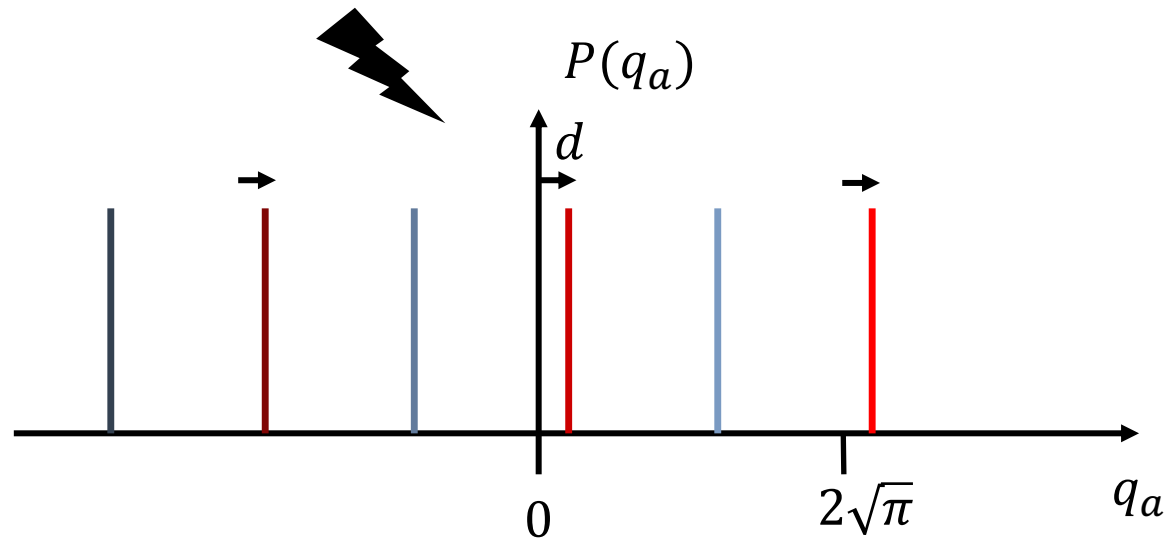
PCI *et al.*, Nature 2020

Sivak *et al.*, Nature 2023

Nord Quantique, PRL 2024



$$T_{CD} = \frac{\sqrt{\pi}}{\chi}$$

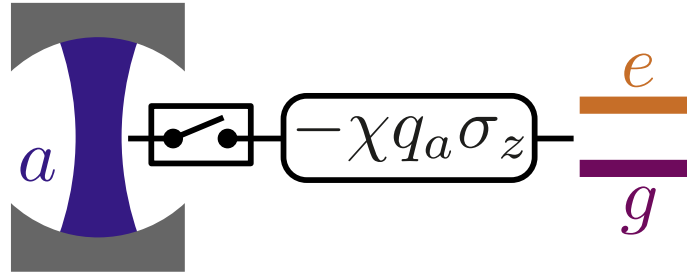


Modular interactions from low-order interactions

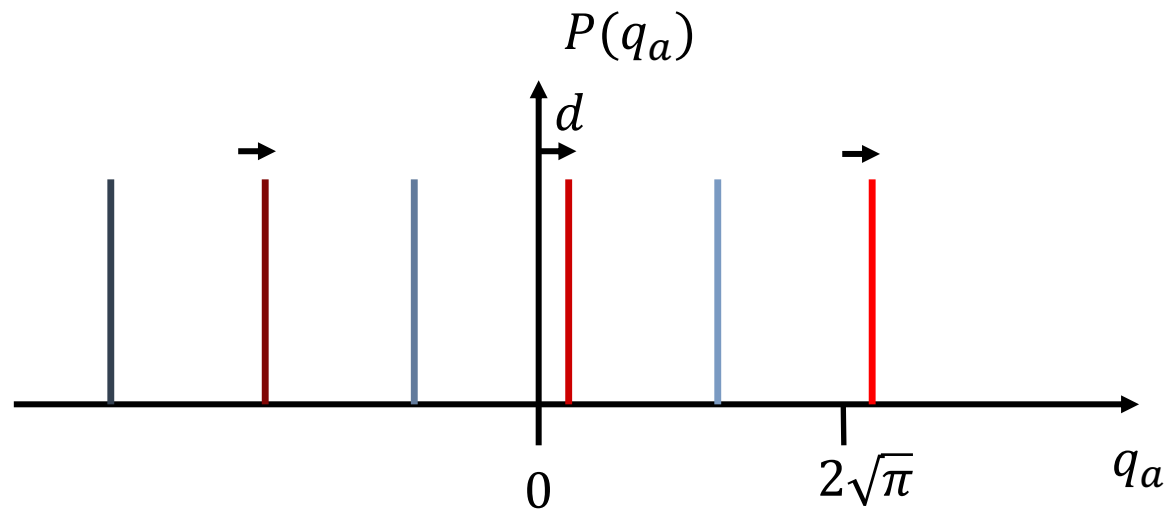
PCI *et al.*, Nature 2020

Sivak *et al.*, Nature 2023

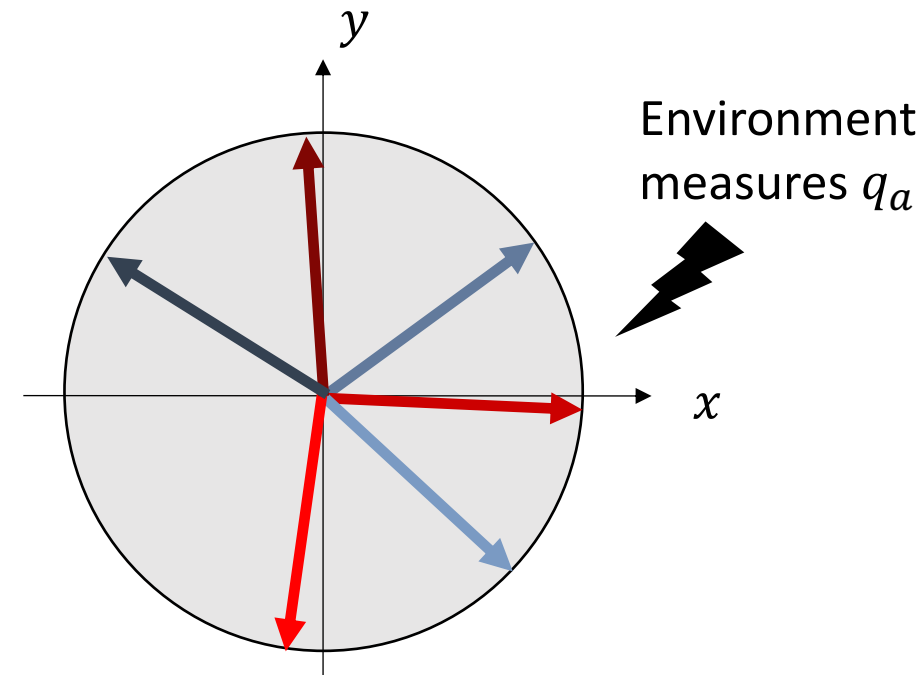
Nord Quantique, PRL 2024



$$T_{CD} = \frac{\sqrt{\pi}}{\chi}$$



Physical qubit errors $\longrightarrow$ GKP qubit errors

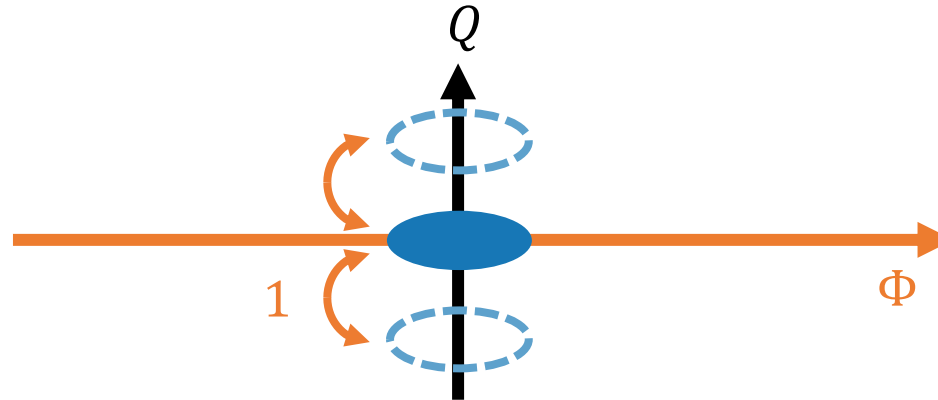
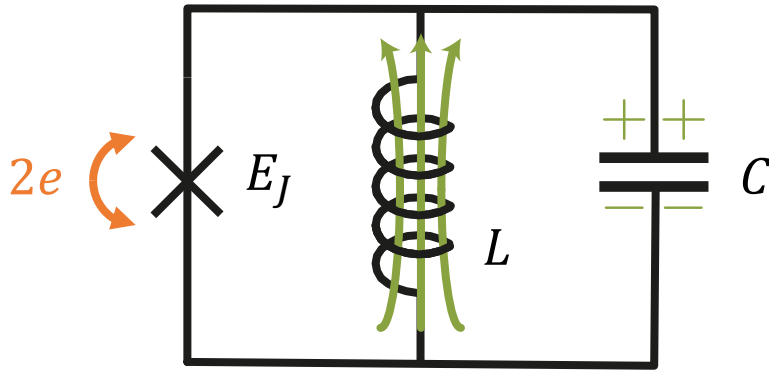


Hamiltonian stabilization of GKP qubits

Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}\mathbf{q}) - E \cos(2\sqrt{\pi}\mathbf{p})$

$$\mathbf{H}_0 = \frac{1}{2C} \mathbf{Q}^2 + \frac{1}{2L} \Phi^2 - E_J \cos(\Phi/\varphi_0)$$

$$[\Phi, \mathbf{Q}] = i\hbar$$



Hamiltonian stabilization of GKP qubits

Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}\mathbf{q}) - E \cos(2\sqrt{\pi}\mathbf{p})$

$$\mathbf{H}_0 = \frac{1}{2} \omega (\mathbf{q}_0^2 + \mathbf{p}_0^2) - E_J \cos(\eta \mathbf{q}_0)$$

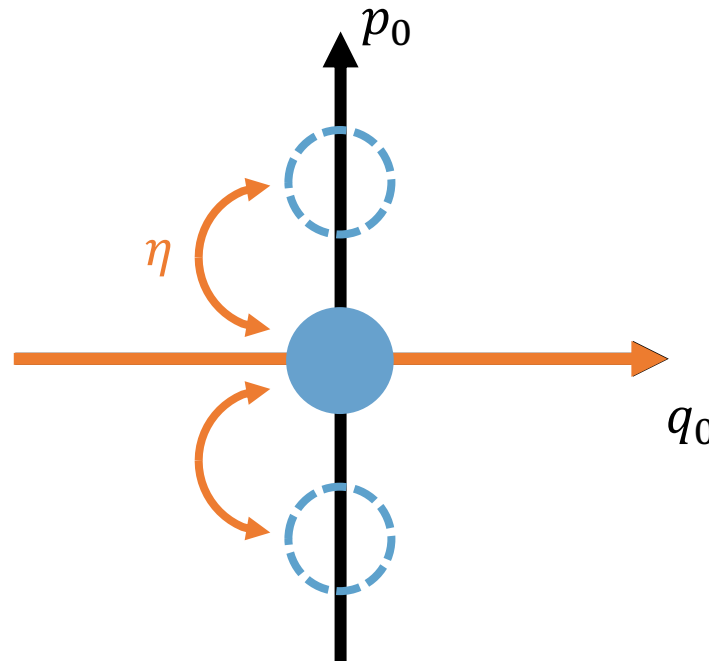
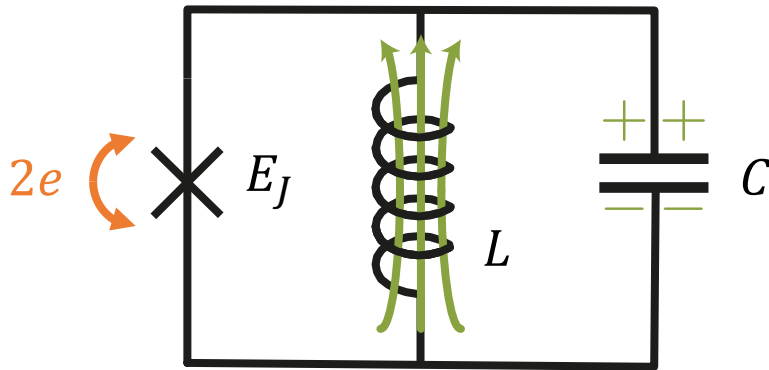
$$[\mathbf{q}_0, \mathbf{p}_0] = i$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\eta = \sqrt{2\pi Z / R_q}$$

$$Z = \sqrt{L/C}$$

$$R_q = 6.5 \text{ k}\Omega$$



Hamiltonian stabilization of GKP qubits

Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}q) - E \cos(2\sqrt{\pi}p)$

$$H(t) = -E_J \cos(\eta \mathbf{q_o}(t))$$

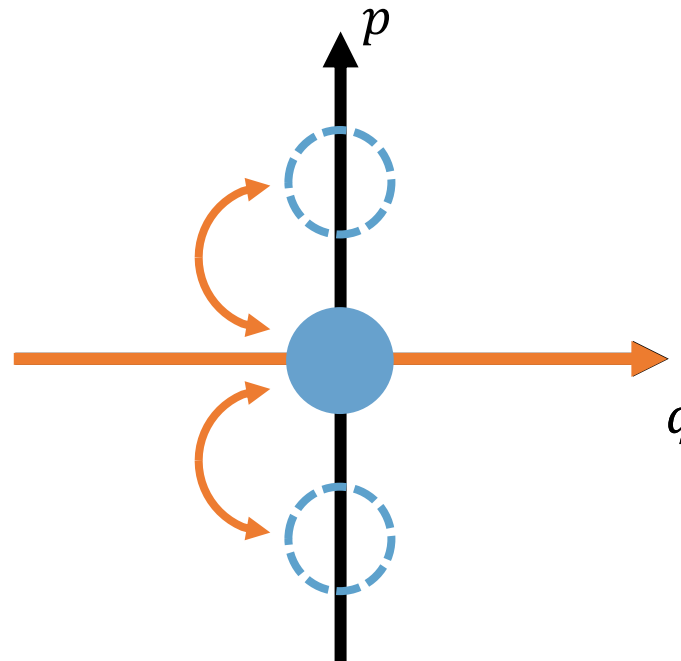
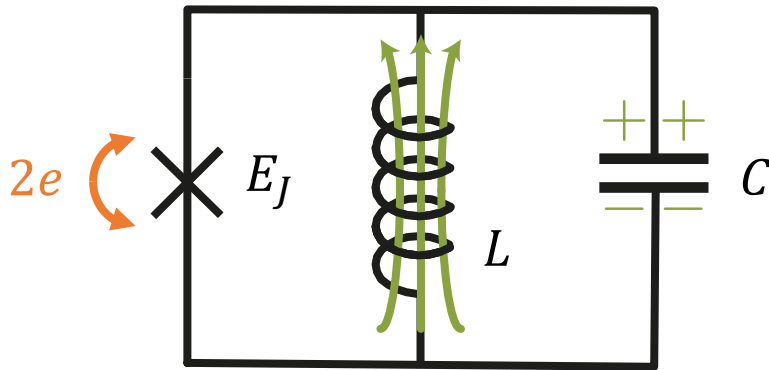
$$[q, p] = i$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\eta = \sqrt{2\pi Z/R_q}$$

$$Z = \sqrt{L/C}$$

$$R_q = 6.5 \text{ k}\Omega$$



Rotating frame

Hamiltonian stabilization of GKP qubits

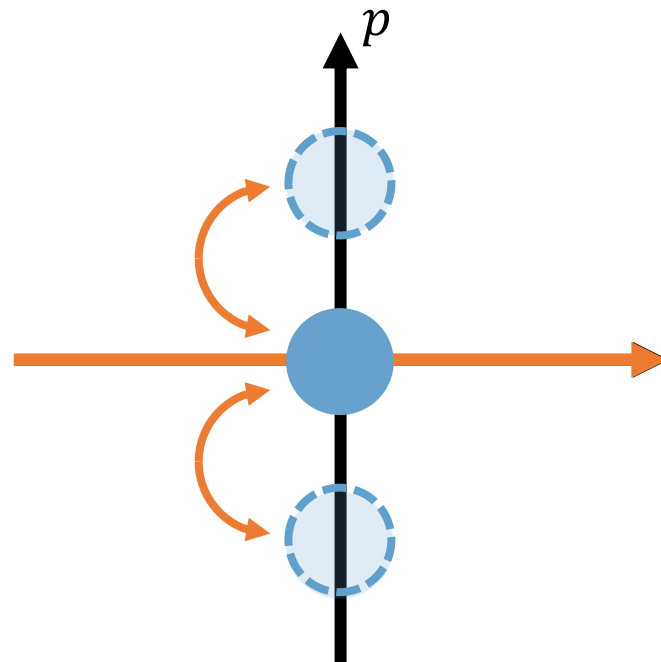
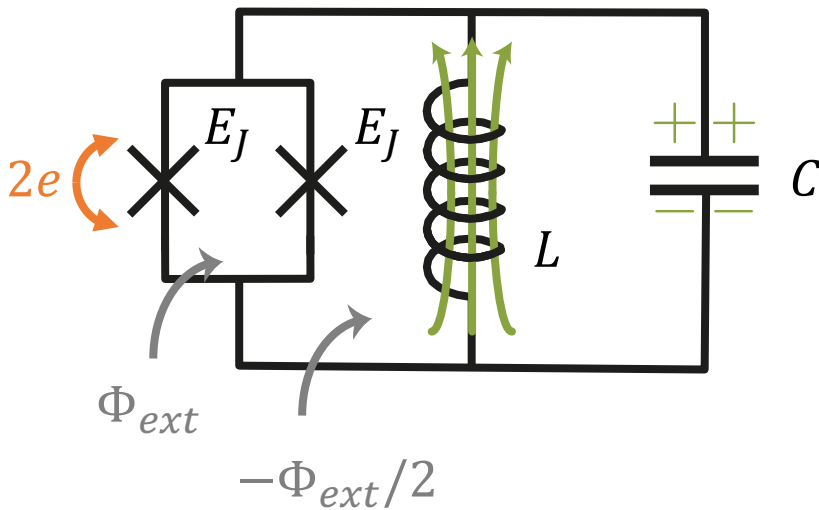
Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}q) - E \cos(2\sqrt{\pi}p)$

$$\mathbf{H}(t) = -E(t) \cos(\eta \mathbf{q}_0(t))$$

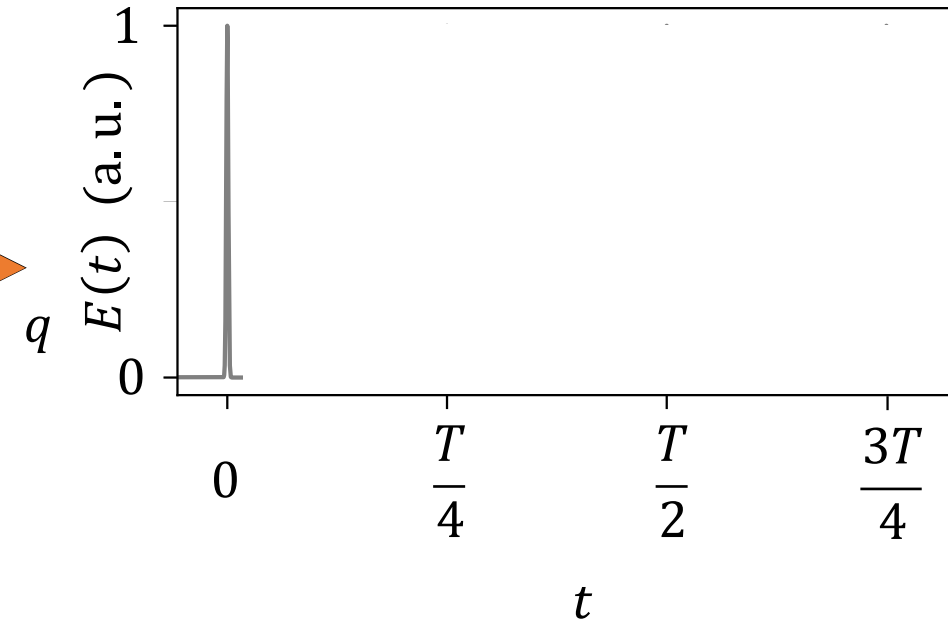
$$[q, p] = i$$

$$\omega = \frac{1}{\sqrt{LC}} \quad \eta = \sqrt{2\pi Z/R_q}$$

$$Z = \sqrt{L/C} \quad R_q = 6.5 \text{ k}\Omega$$



Rotating frame



Hamiltonian stabilization of GKP qubits

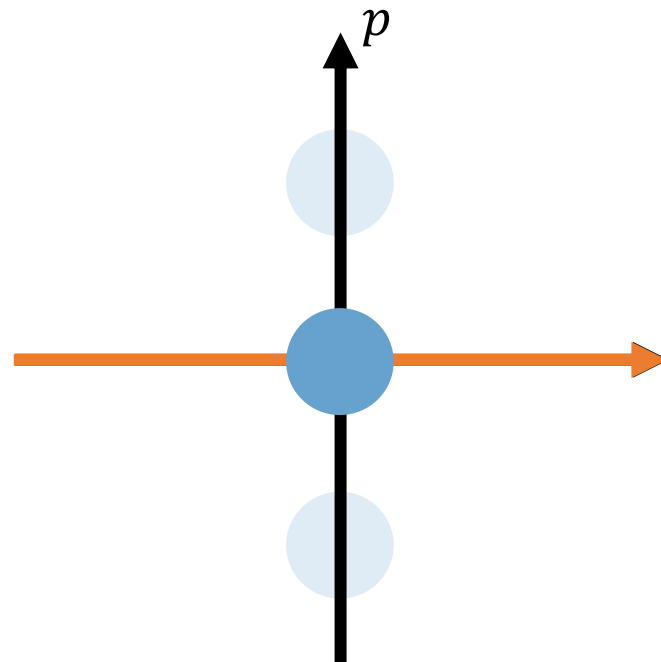
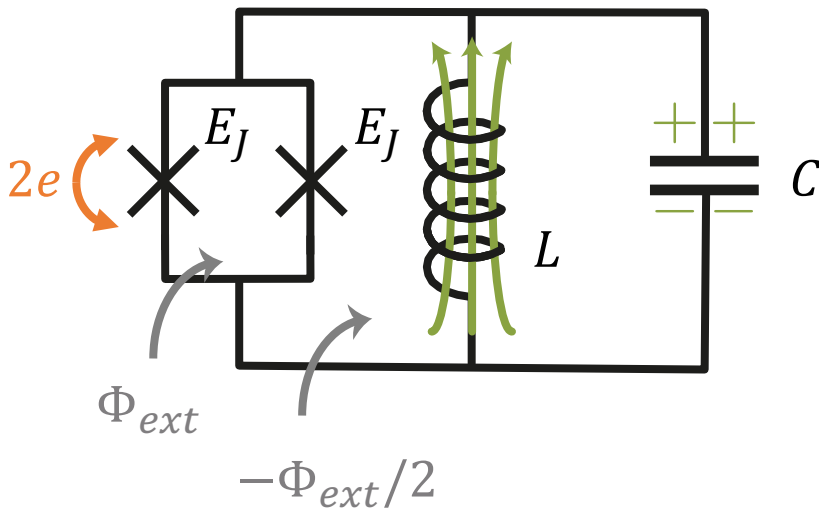
Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}q) - E \cos(2\sqrt{\pi}p)$

$$\mathbf{H}(t) = -E(t) \cos(\eta \mathbf{q}_0(t))$$

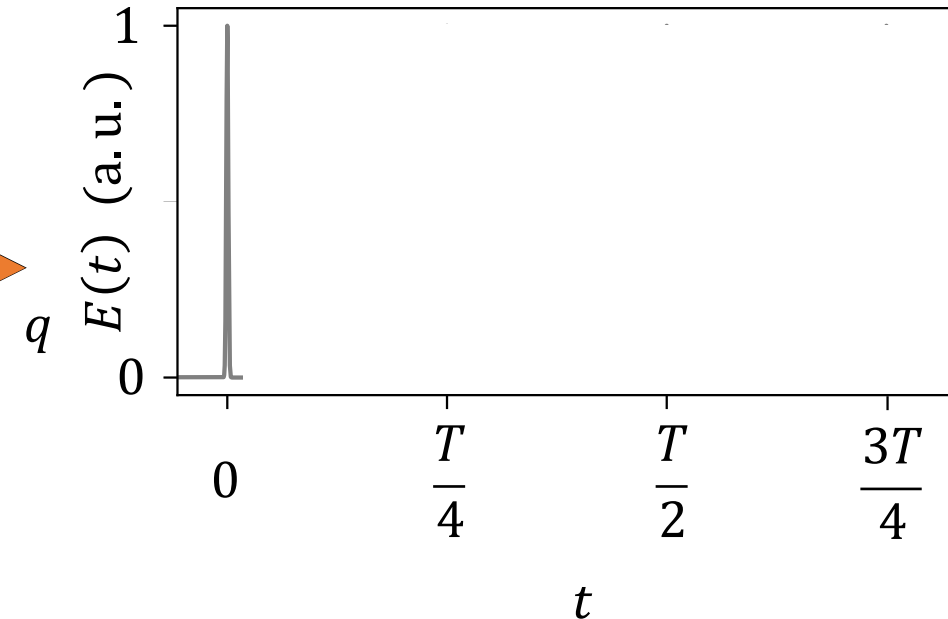
$$[q, p] = i$$

$$\omega = \frac{1}{\sqrt{LC}} \quad \eta = \sqrt{2\pi Z / R_q}$$

$$Z = \sqrt{L/C} \quad R_q = 6.5 \text{ k}\Omega$$



Rotating frame



Hamiltonian stabilization of GKP qubits

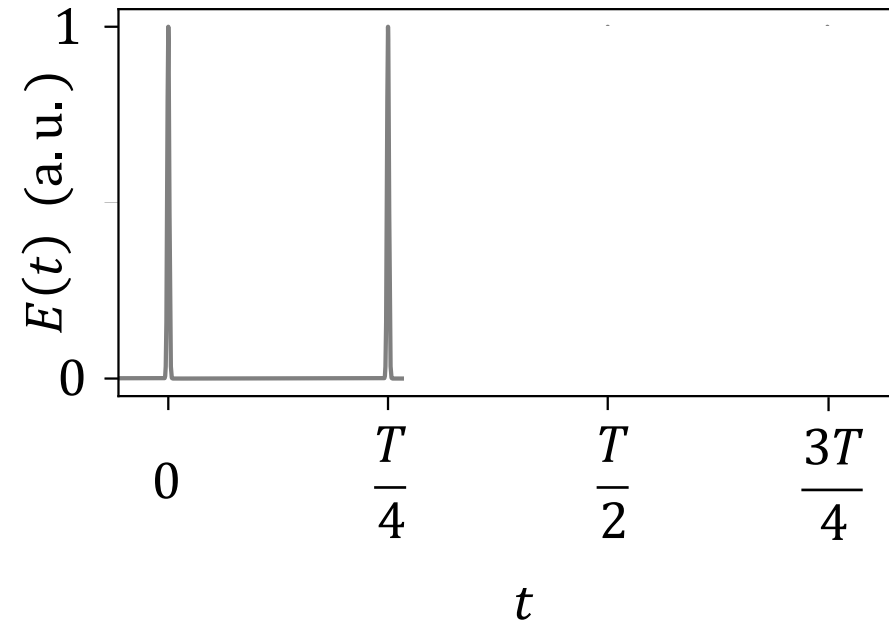
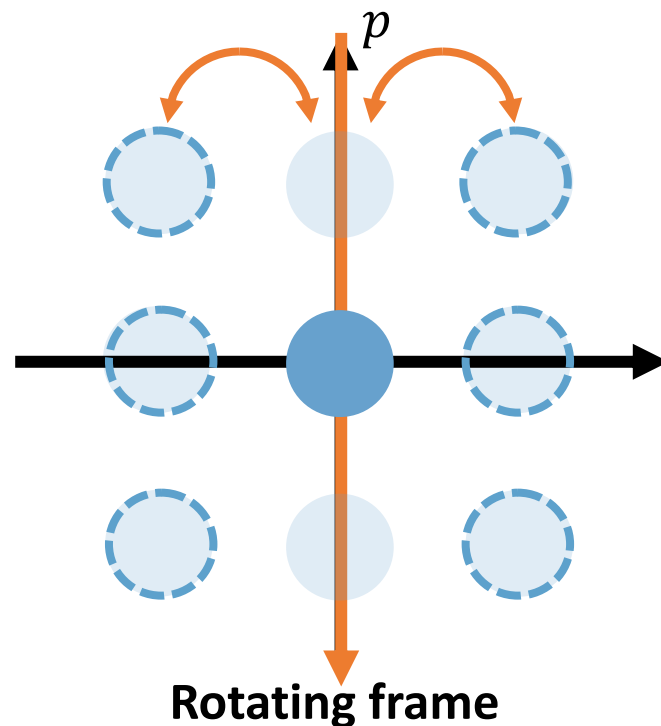
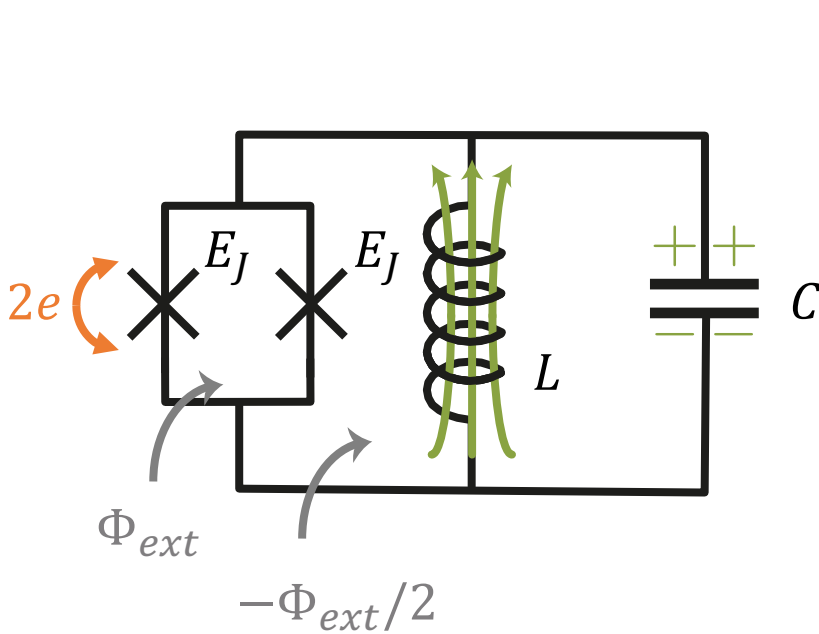
Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}q) - E \cos(2\sqrt{\pi}p)$

$$\mathbf{H}(t) = -E(t) \cos(\eta \mathbf{q}_0(t))$$

$$[q, p] = i$$

$$\omega = \frac{1}{\sqrt{LC}} \quad \eta = \sqrt{2\pi Z / R_q}$$

$$Z = \sqrt{L/C} \quad R_q = 6.5 \text{ k}\Omega$$



Hamiltonian stabilization of GKP qubits

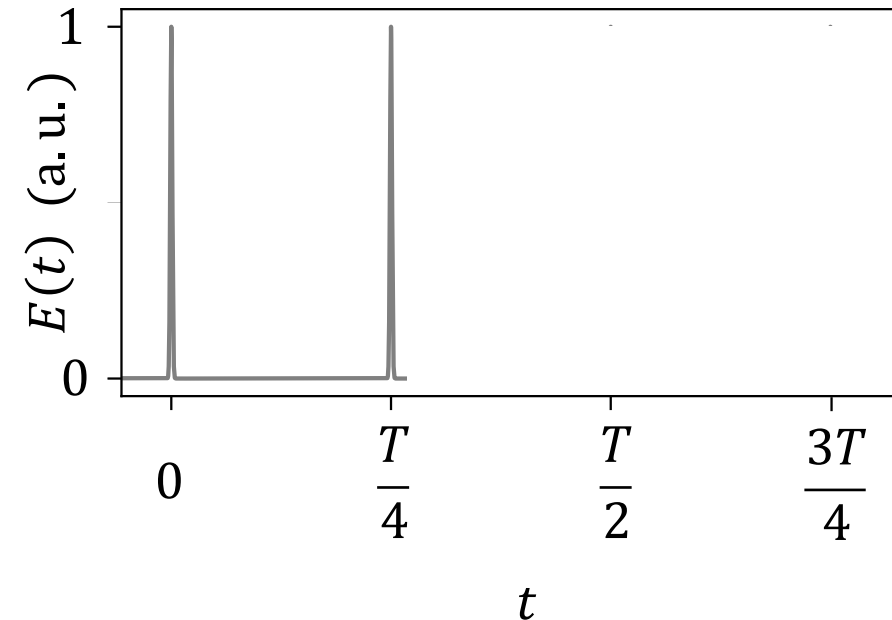
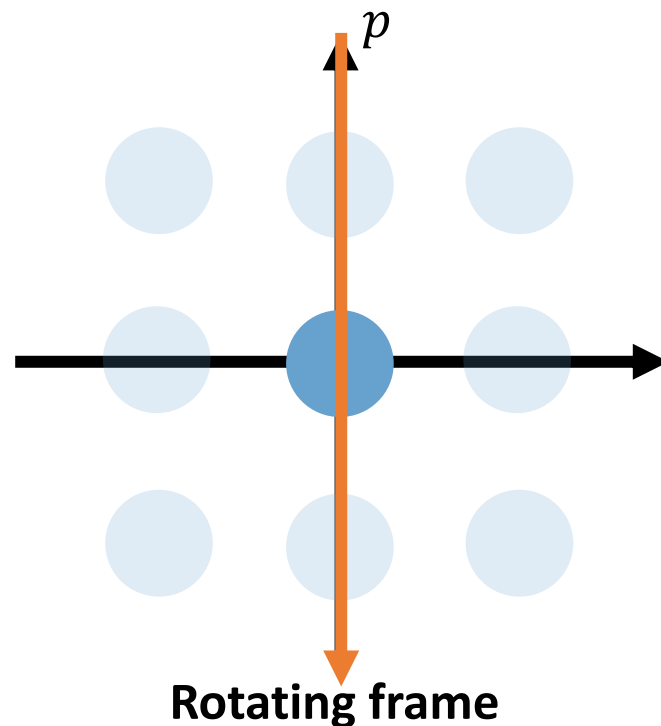
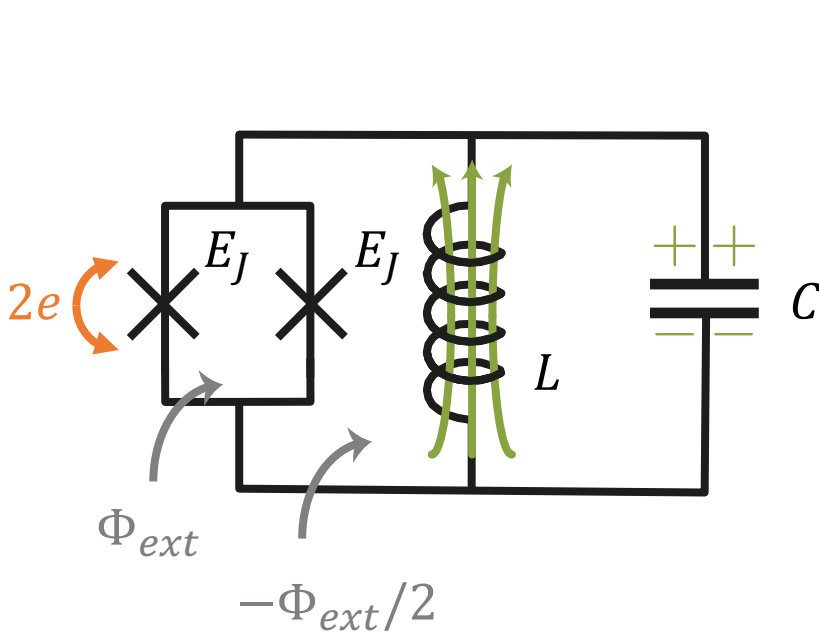
Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}\mathbf{q}) - E \cos(2\sqrt{\pi}\mathbf{p})$

$$\mathbf{H}(t) = -E(t) \cos(\eta \mathbf{q}_0(t))$$

$$[\mathbf{q}, \mathbf{p}] = i$$

$$\omega = \frac{1}{\sqrt{LC}} \quad \eta = \sqrt{2\pi Z / R_q}$$

$$Z = \sqrt{L/C} \quad R_q = 6.5 \text{ k}\Omega$$



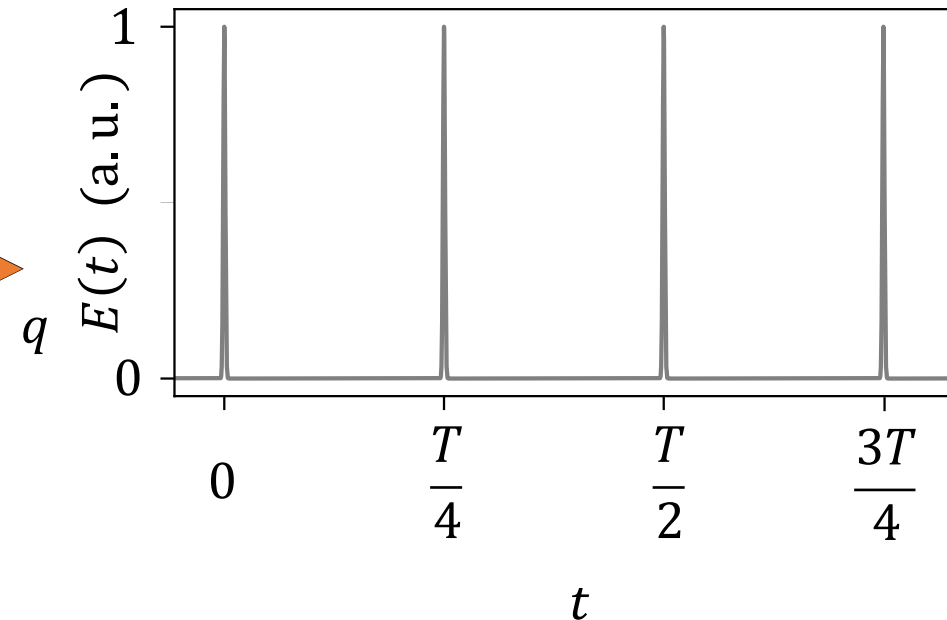
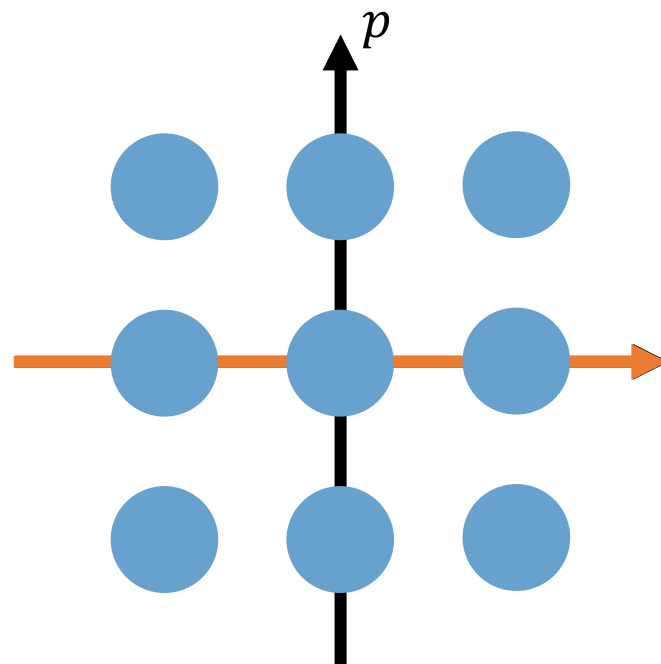
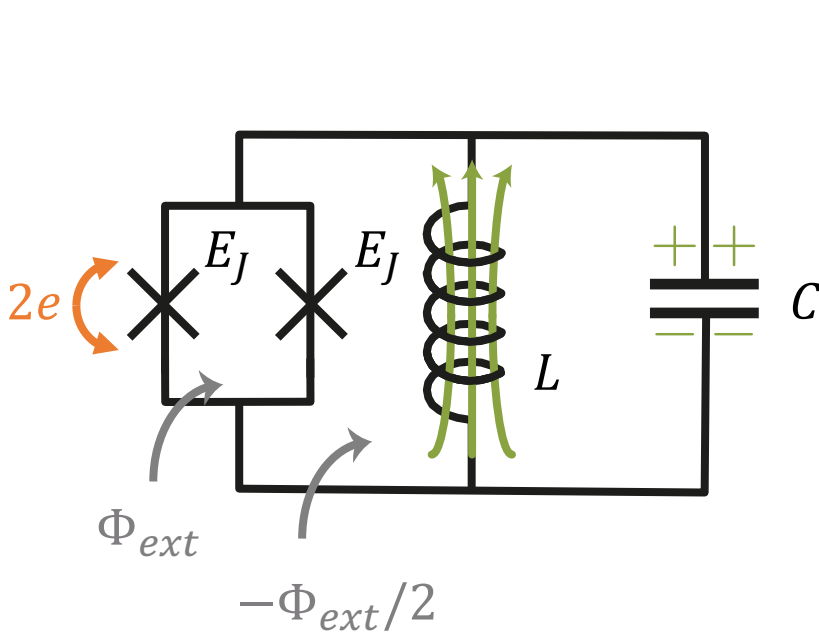
Hamiltonian stabilization of GKP qubits

Goal : $\mathbf{H}_{GKP} = -E \cos(2\sqrt{\pi}q) - E \cos(2\sqrt{\pi}p)$

$$\mathbf{H}_{eff} = \overline{\mathbf{H}(t)} = -\frac{\overline{E(t)}}{2} (\cos(\eta q) + \cos(\eta p))$$

$$\eta = \sqrt{2\pi Z/R_q}$$

$$Z = 2 R_q$$

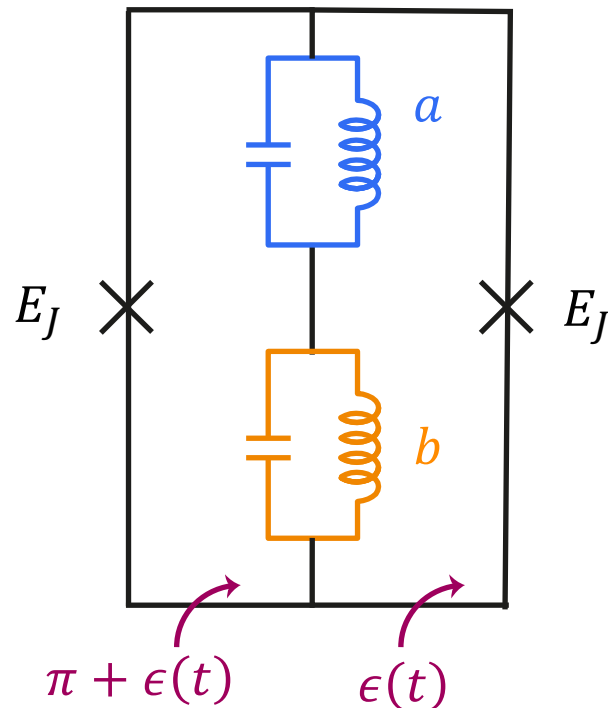


Modular interaction engineering

Goal : $H_{mod} = g \cos(2\sqrt{\pi}q_a) q_b$

$$H(t) = 2E_J \sin(\epsilon(t)) \sin(\eta_a q_a^0(t) + \eta_b q_b^0(t))$$

Equivalent ATS circuit

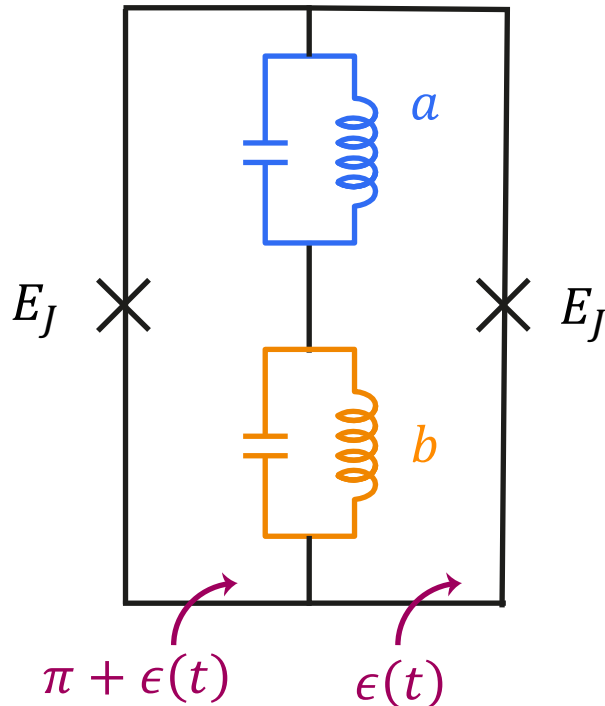


Modular interaction engineering

Goal : $H_{mod} = g \cos(2\sqrt{\pi}q_a) q_b$

$$H(t) = E(t) \begin{pmatrix} \sin(\eta_a q_a^0(t)) \cos(\eta_b q_b^0(t)) + \\ \cos(\eta_a q_a^0(t)) \sin(\eta_b q_b^0(t)) \end{pmatrix}$$

Equivalent ATS circuit

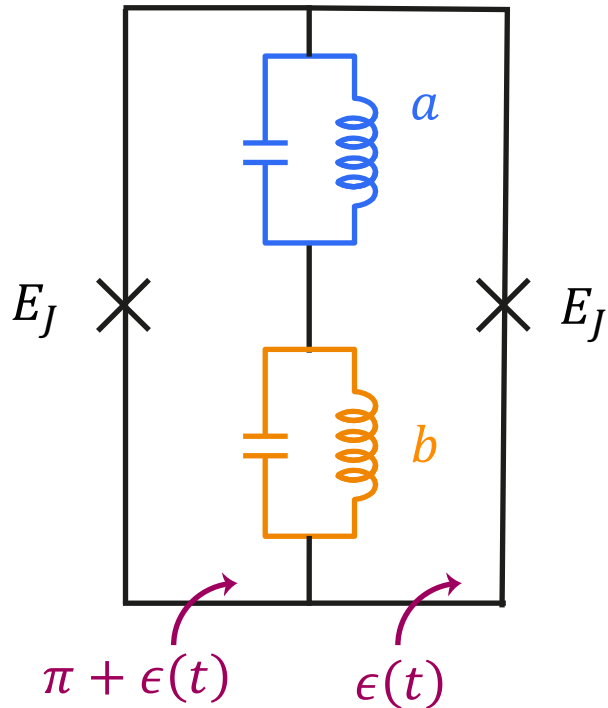


Modular interaction engineering

Goal : $H_{mod} = g \cos(2\sqrt{\pi}q_a) q_b$

$$H(t) = E(t) \begin{pmatrix} \sin(\eta_a q_a^0(t)) + \\ \cos(\eta_a q_a^0(t)) \eta_b q_b^0(t) \end{pmatrix}$$

Equivalent ATS circuit



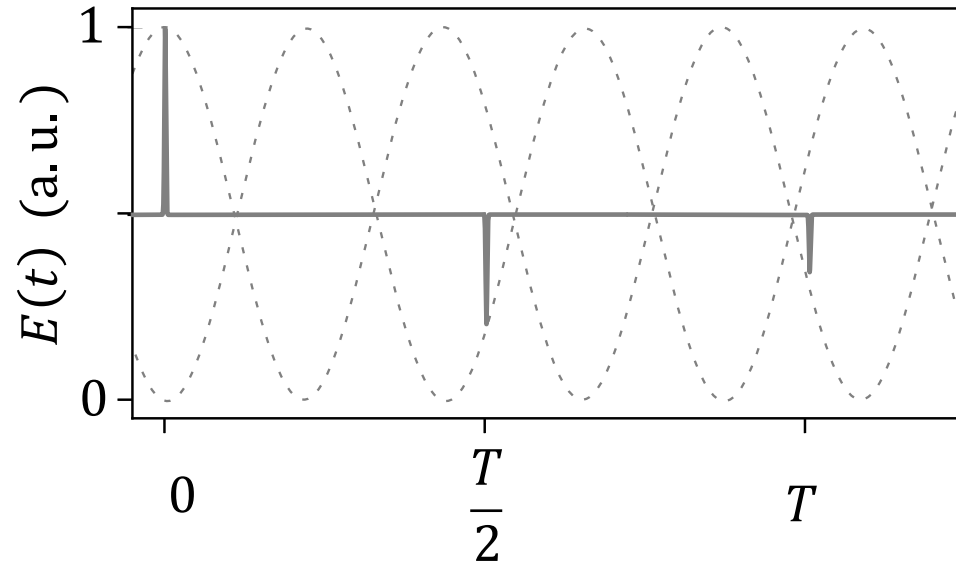
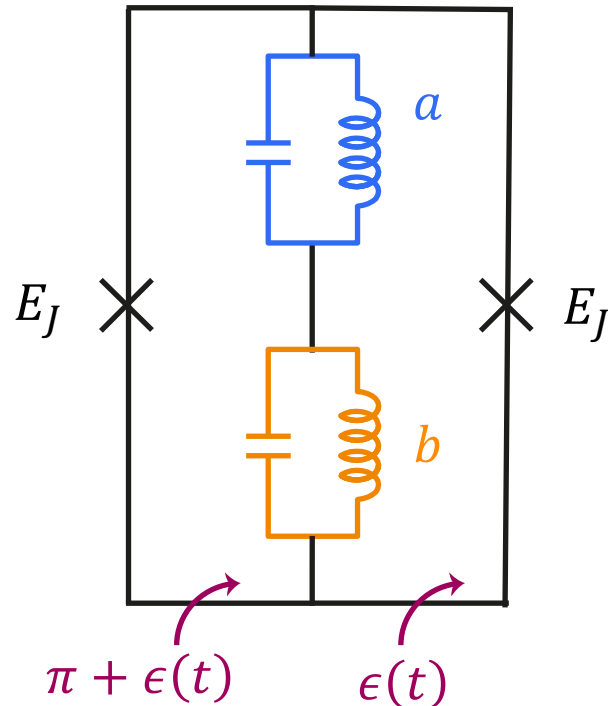
$$\eta_b \ll 1$$

Modular interaction engineering

Goal : $H_{mod} = g \cos(2\sqrt{\pi}q_a) q_b$

$$H(t) = E(t) \begin{pmatrix} \sin(\eta_a q_a^0(t)) + \\ \cos(\eta_a q_a^0(t)) \eta_b q_b^0(t) \end{pmatrix}$$

Equivalent ATS circuit



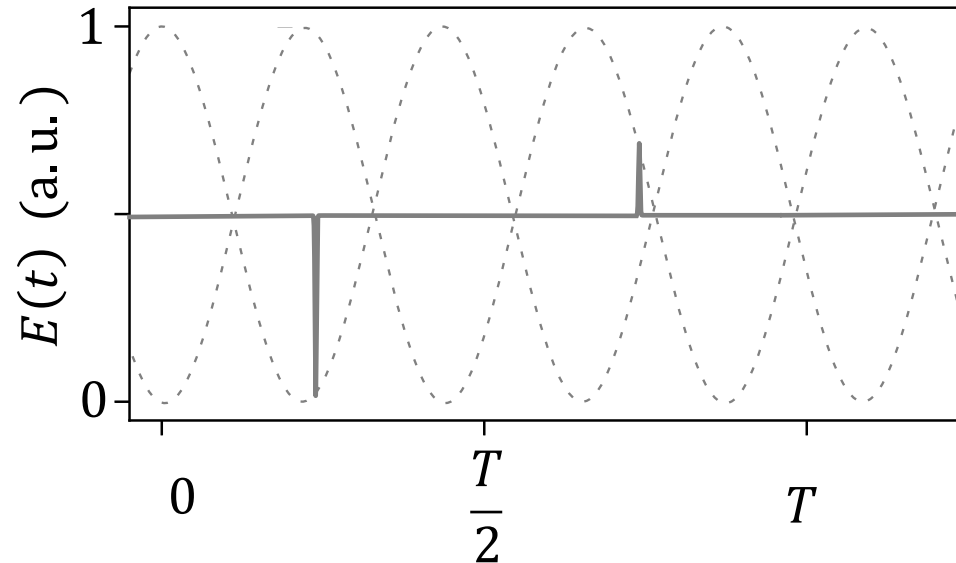
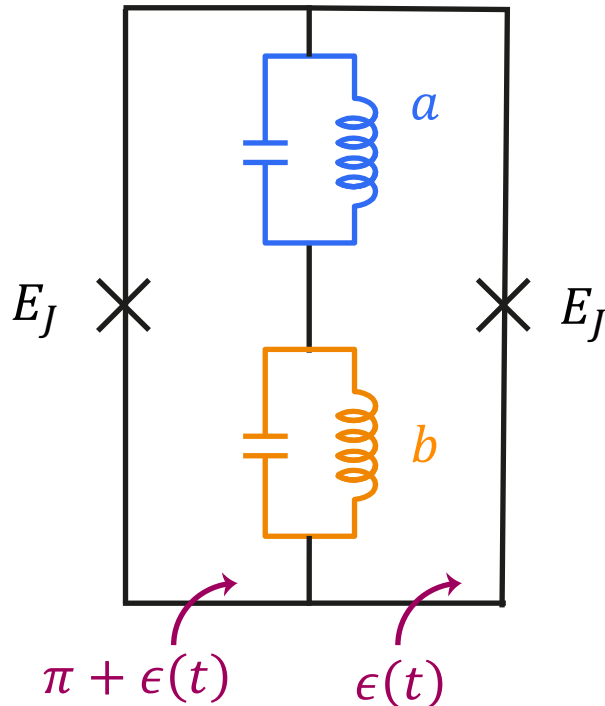
$$\eta_b \ll 1 \xrightarrow{\text{RWA}} \overline{H(t)} = H_{mod}$$

Modular interaction engineering

Goal : $H_{mod} = g \cos(2\sqrt{\pi}p_a) q_b$

$$H(t) = E(t) \begin{pmatrix} \cos(\eta_a q_a^0(t)) + \\ \sin(\eta_a q_a^0(t)) \eta_b q_b^0(t) \end{pmatrix}$$

Equivalent ATS circuit

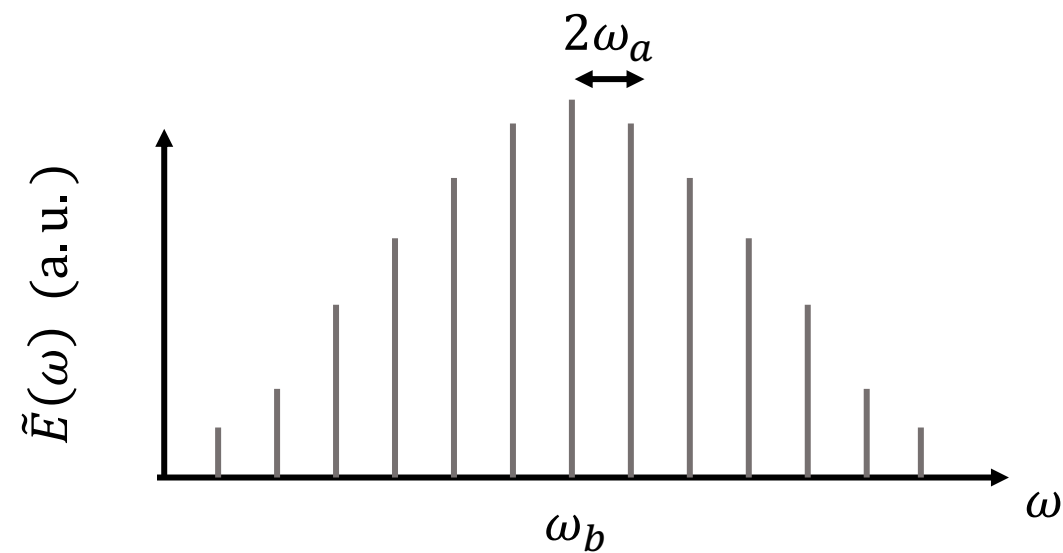
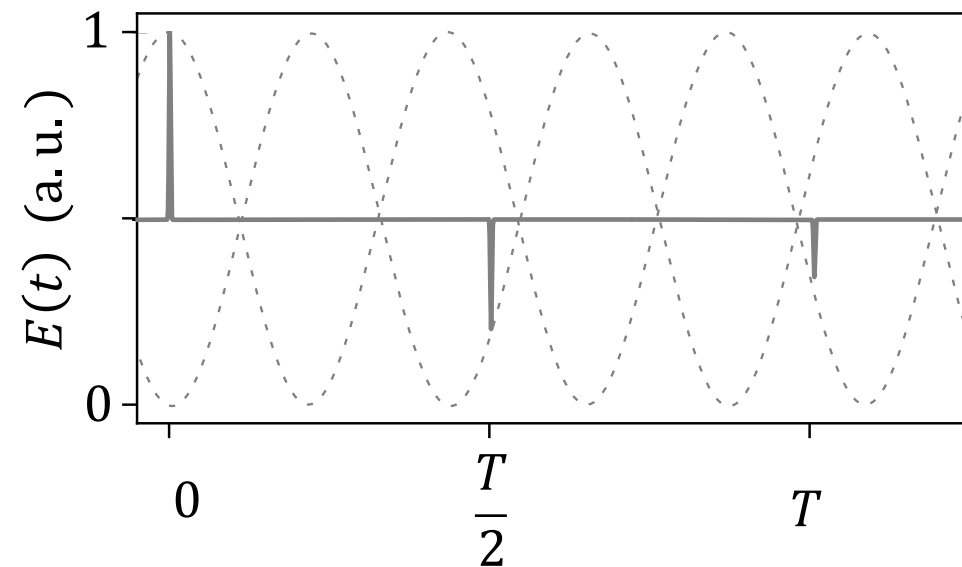
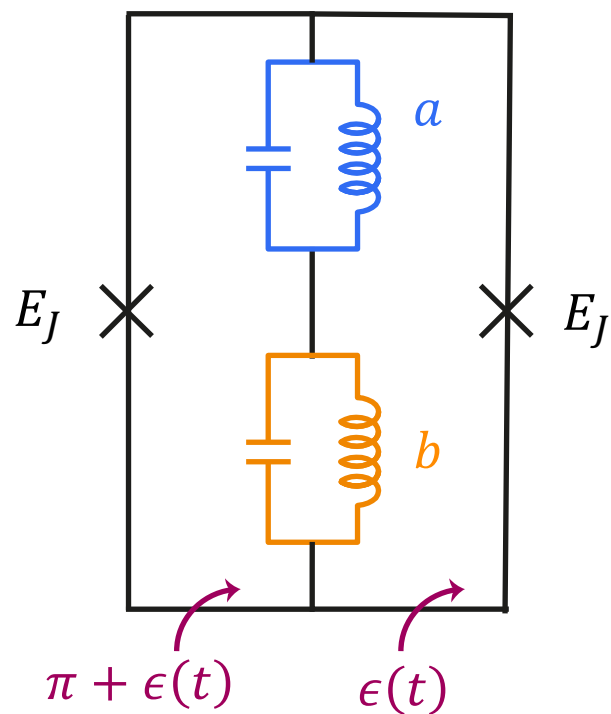


$$\eta_b \ll 1 \xrightarrow{\text{RWA}} \overline{H(t)} = H_{mod}$$

Frequency-comb drive

$$\varphi_{ZPF,a} = \sqrt{2\pi} \quad (Z_a = 13 \text{ k}\Omega)$$

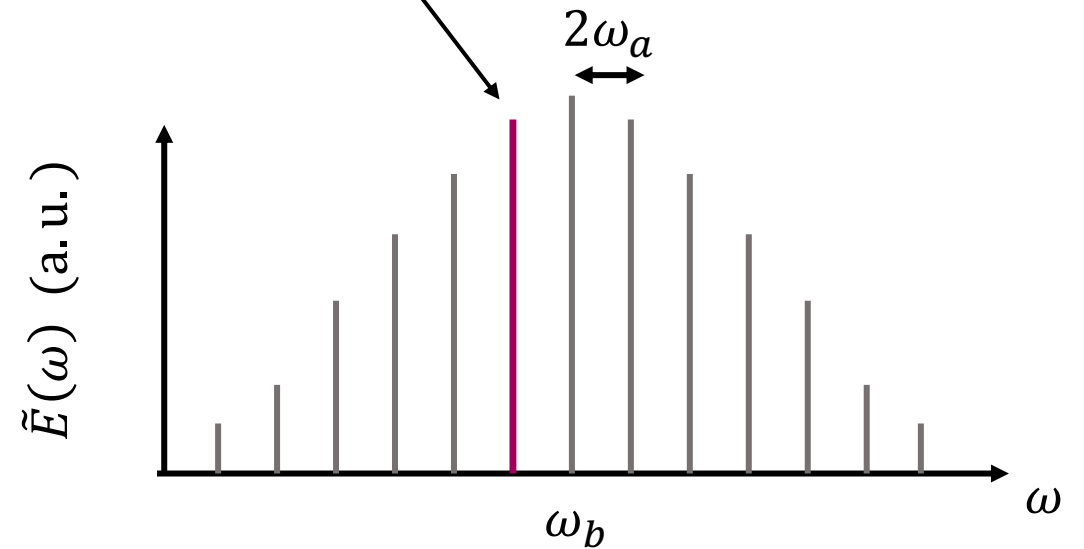
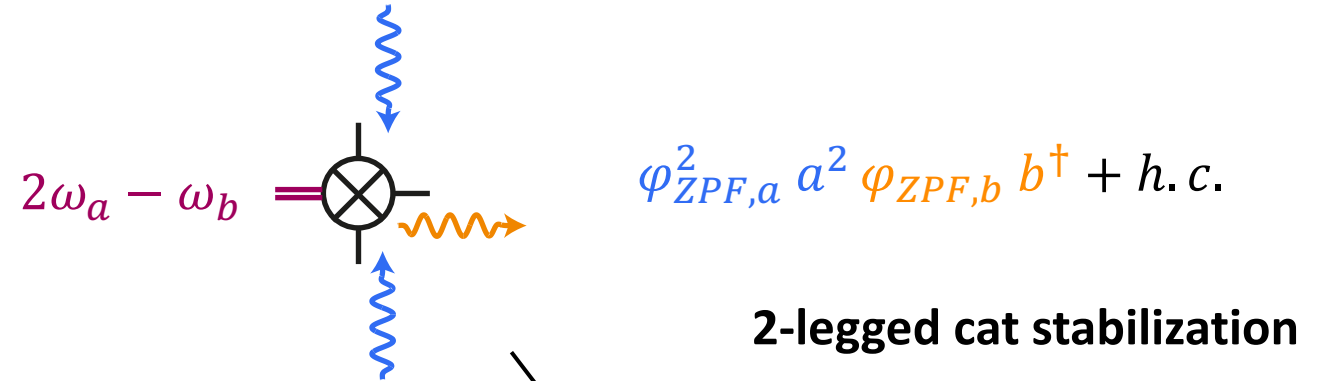
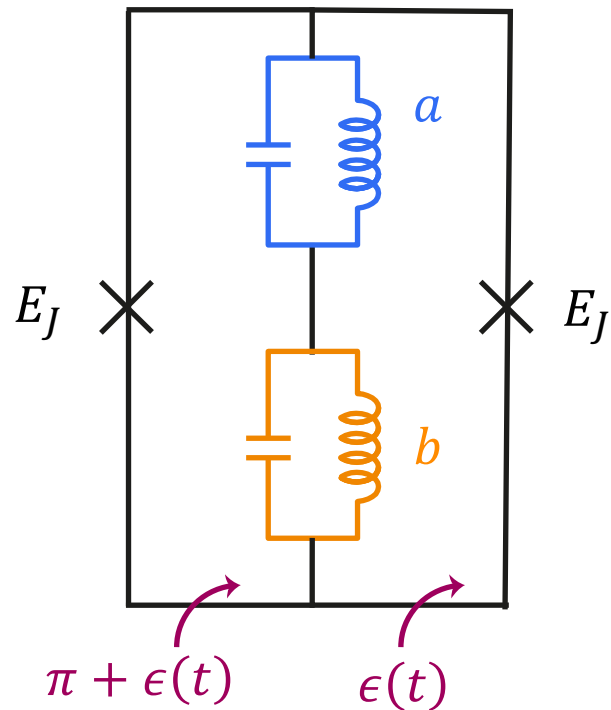
$$\omega_a/2\pi < 1 \text{ GHz}$$



Frequency-comb drive

$$\varphi_{\text{ZPF},a} \sim 0.05$$

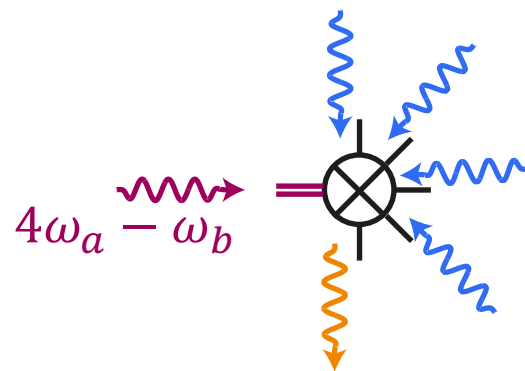
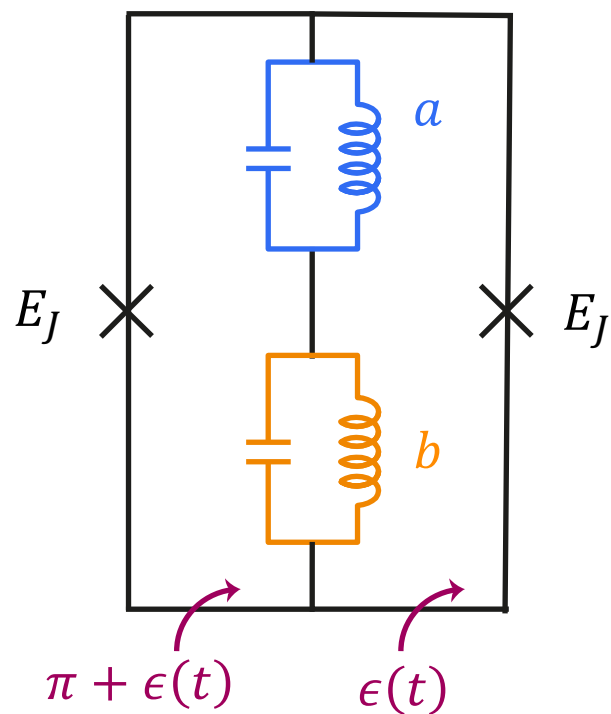
$$\frac{\omega_a}{2\pi} \sim 5 \text{ GHz}$$



Frequency-comb drive

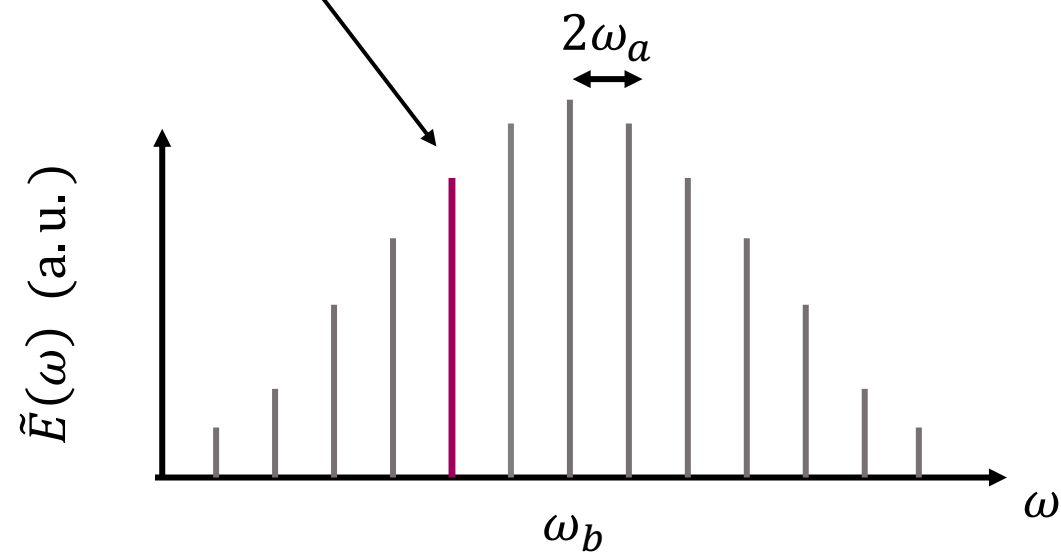
$$\varphi_{ZPF,a} \sim 0.5$$

$$\frac{\omega_a}{2\pi} \sim 5 \text{ GHz}$$



$$\varphi_{ZPF,a}^4 a^4 \varphi_{ZPF,b} b^\dagger + h.c.$$

4-legged cat stabilization



~~5-year plan~~ a.u.

Z_a	500 Ω	1 k Ω	1 k Ω	1.6 k Ω	13 k Ω
$\omega_a/2\pi$	5 GHz	2 GHz	2 GHz	1 GHz	0.3 GHz
N_{harm}	1	1	2 – 4	5 – 10	25 – 50

4-legged cats

N -legged cats
($5 \leq N \leq 8$)

Generalized
cat codes

Rojkov et al., PRX, 2026

Single
GKP state

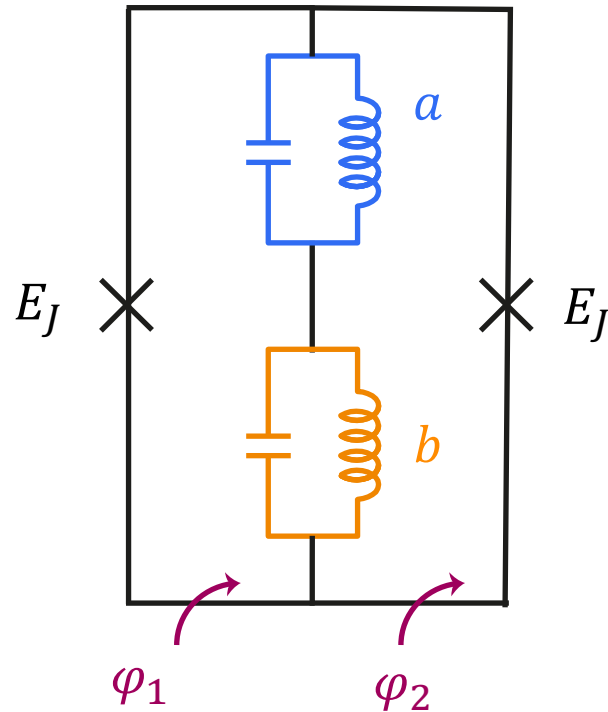
GKP qubits
& gates

Outline

1. Bosonic codes: structure
2. Bosonic codes: dissipative stabilization
3. First experimental steps (4-legged cats)

High-impedance circuit

Equivalent circuit

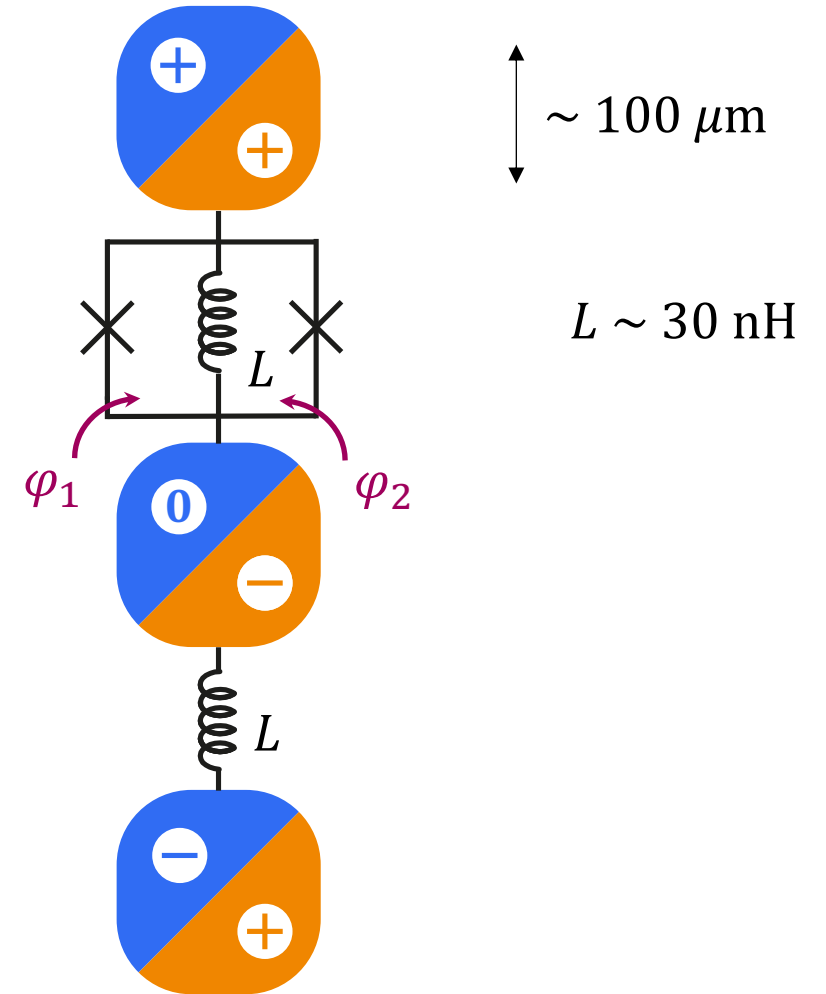


$$\omega_a = 2\pi \times 4.5 \text{ GHz}$$

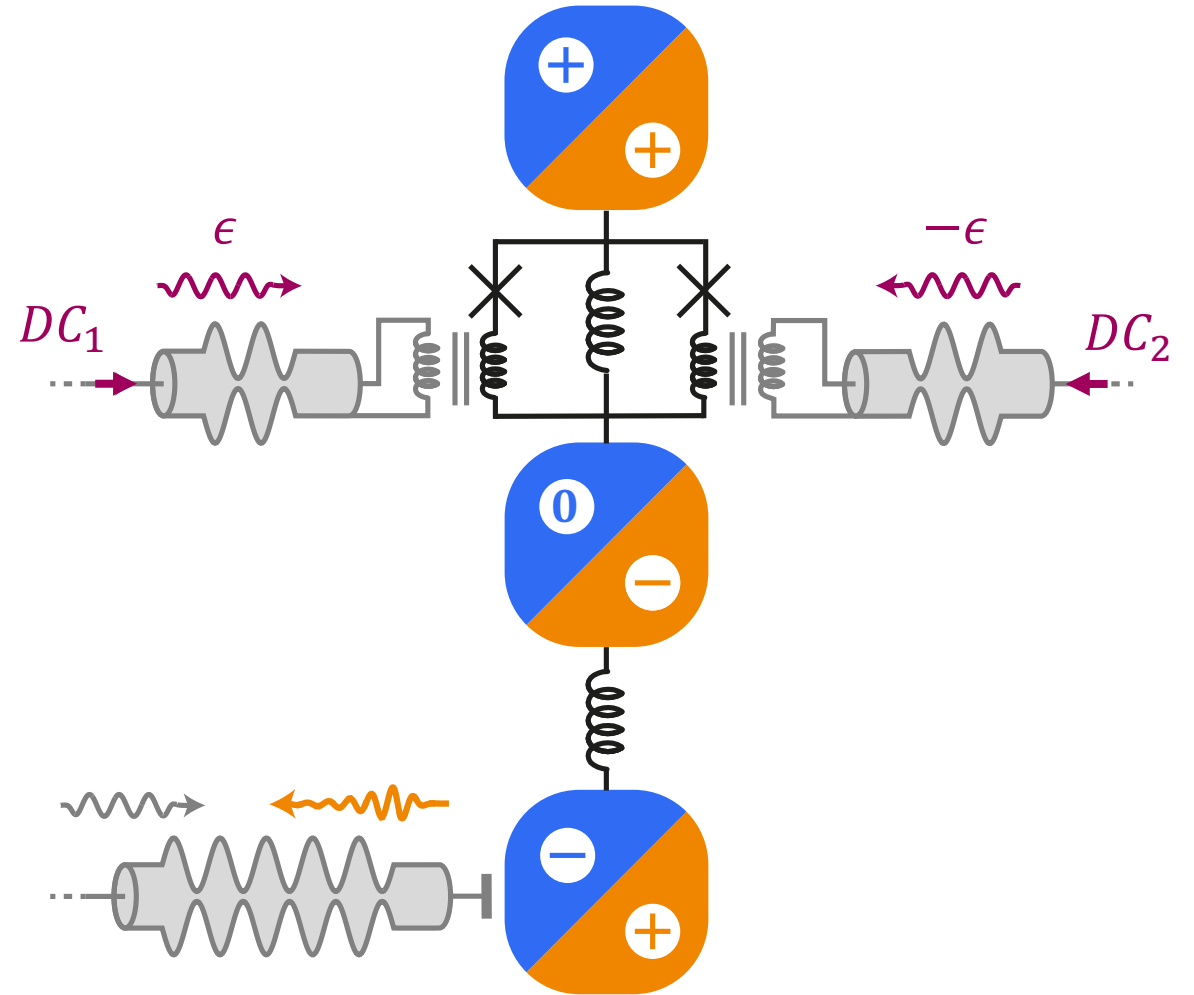
$$\omega_b = 2\pi \times 8 \text{ GHz}$$

$$E_J = h \times 6 \text{ GHz}$$

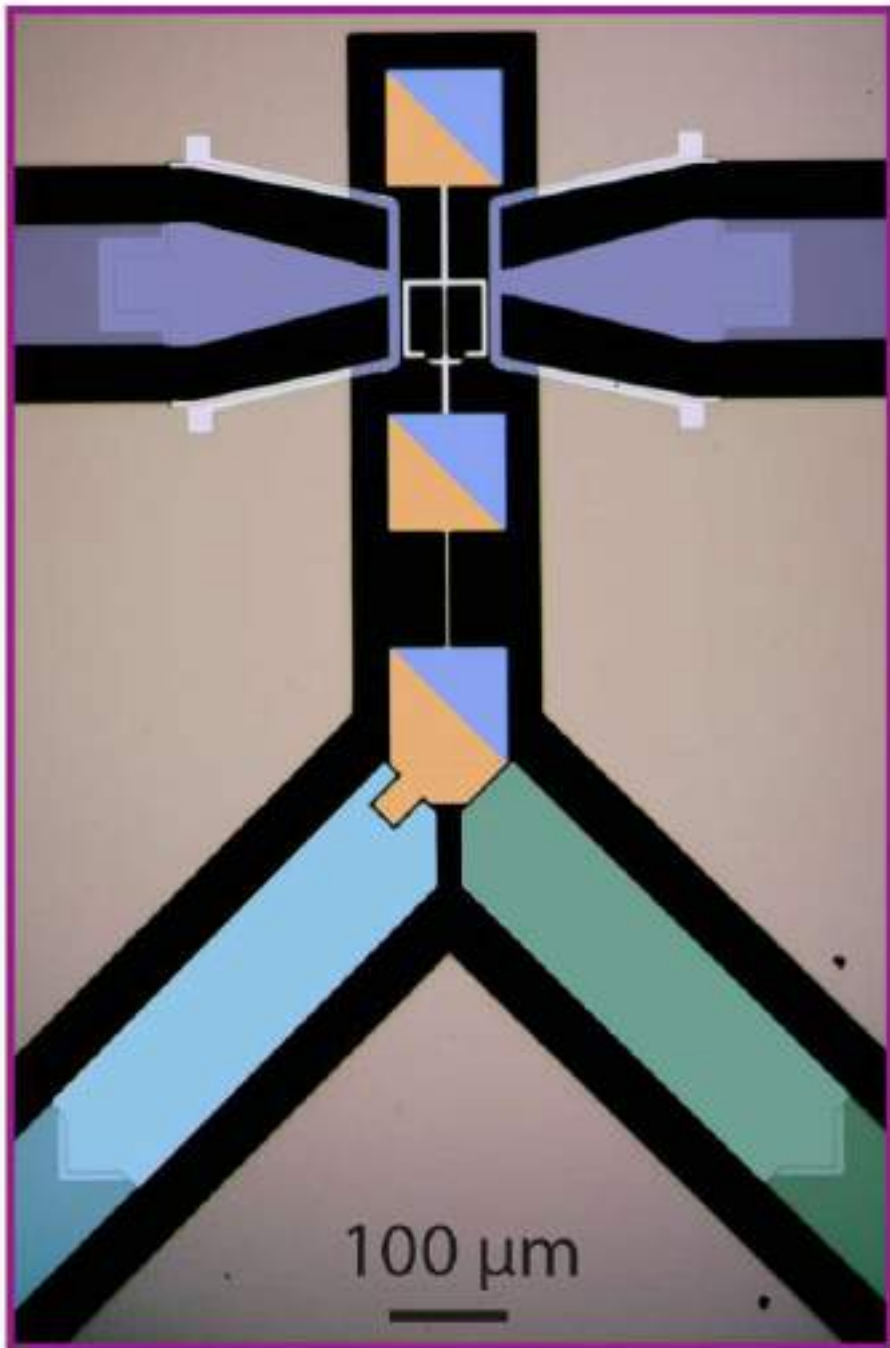
$$\left. \begin{array}{l} \varphi_{ZPF,a} \\ \varphi_{ZPF,b} \end{array} \right\} \sim 0.5$$



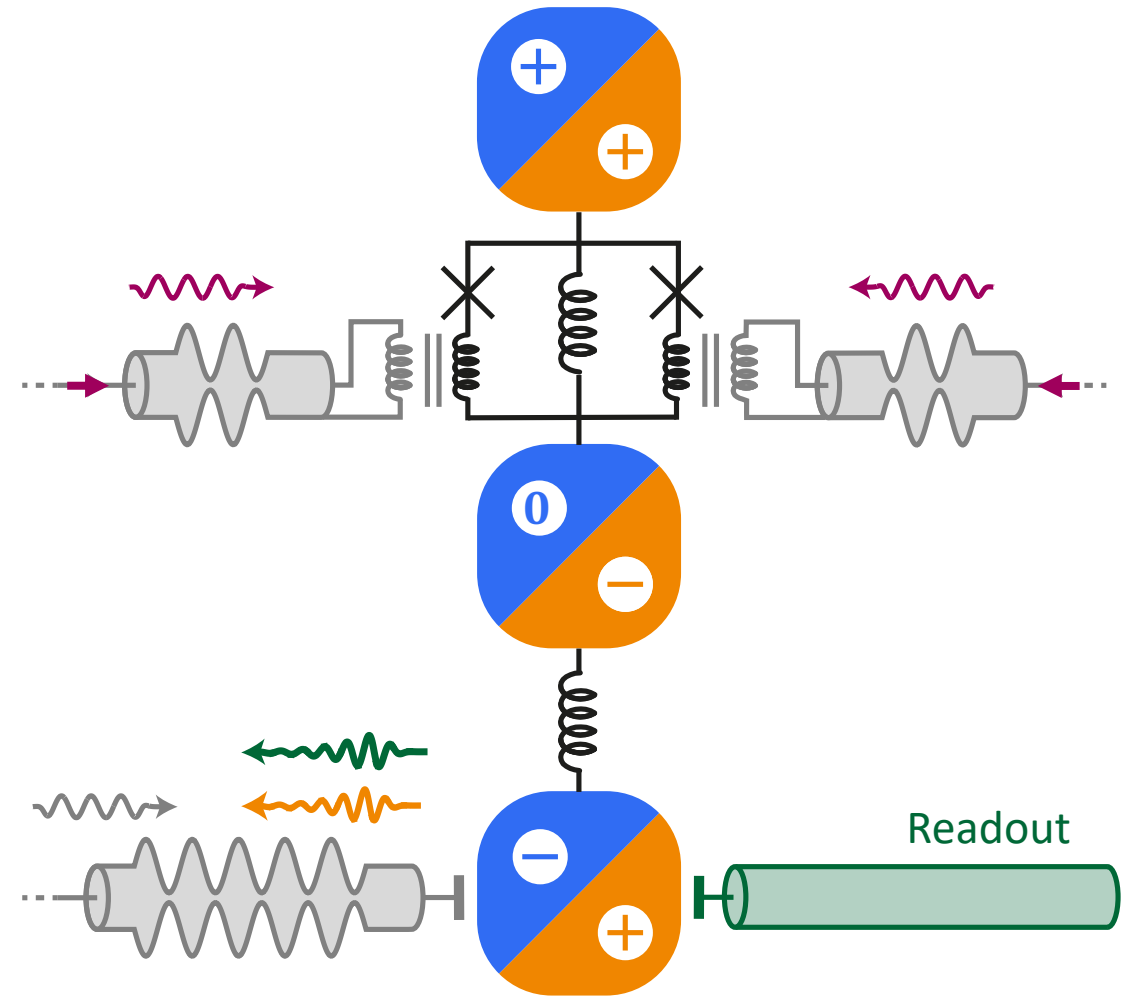
Flux and charge drives



$$\left. \begin{array}{l} \varphi_{ZPF,a} \\ \varphi_{ZPF,b} \end{array} \right\} \sim 0.5$$

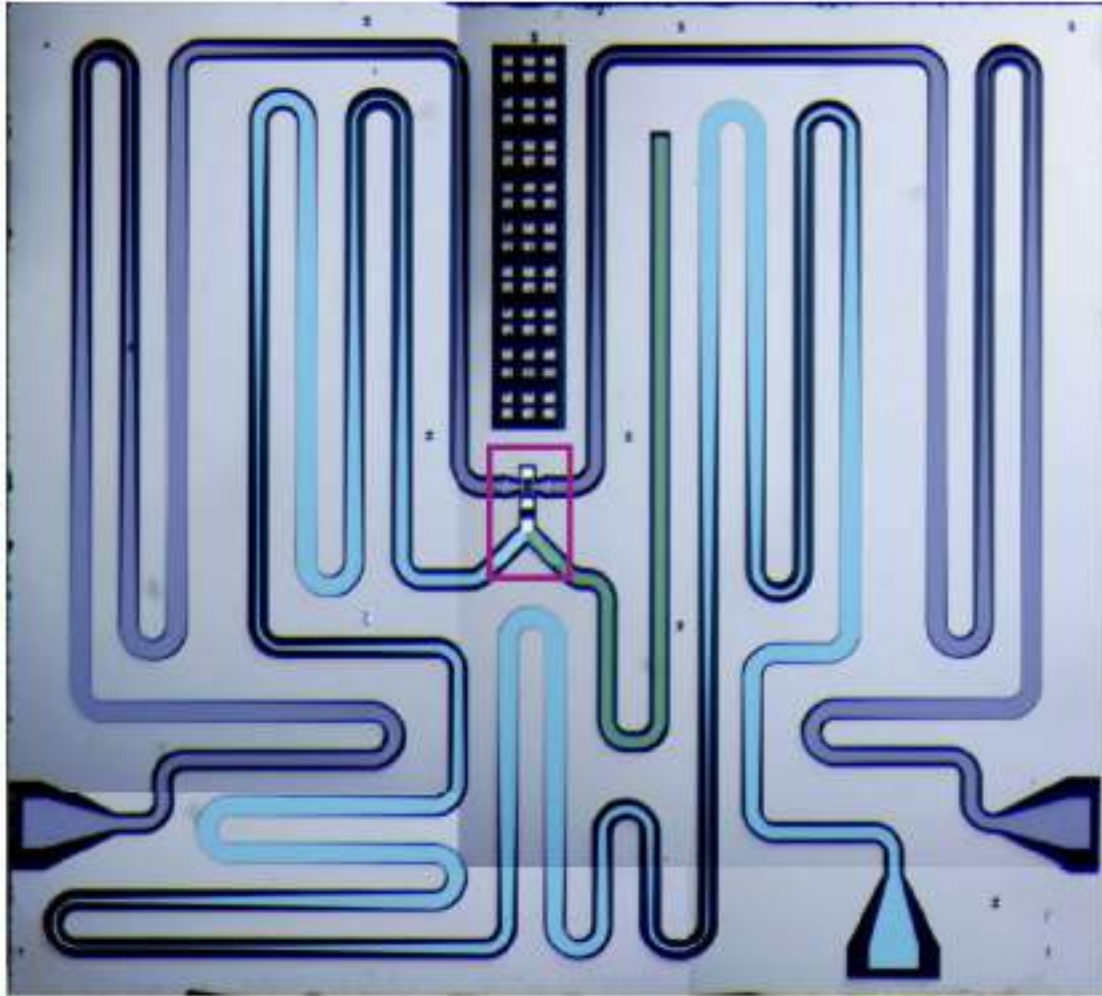


Three-mode circuit

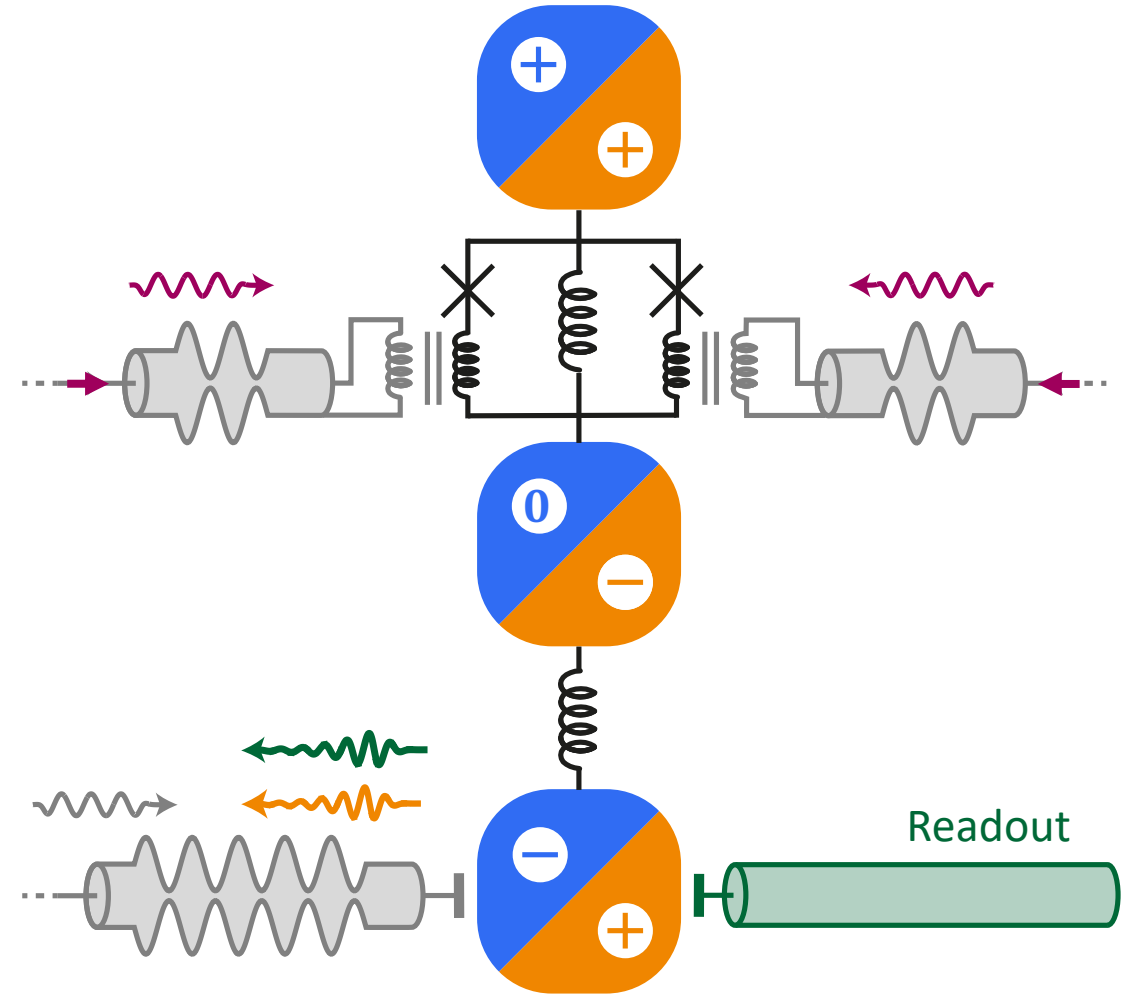


$$\varphi_{\text{ZPF},r} \sim 0.05$$

Three-mode circuit



1 mm

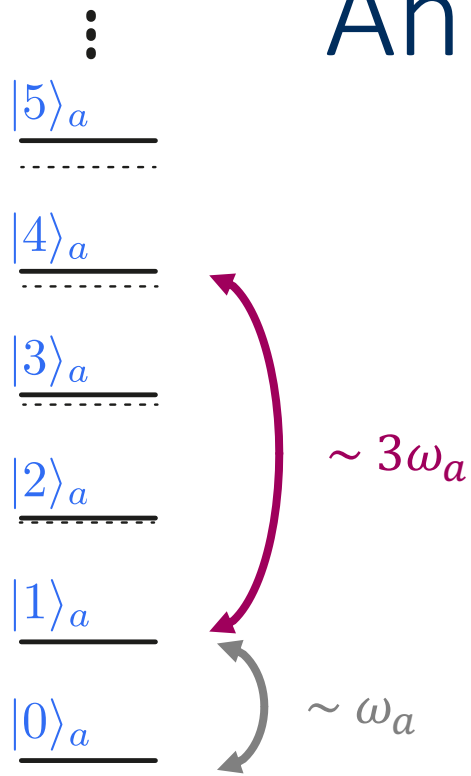


$$\varphi_{\text{ZPF},r} \sim 0.05$$

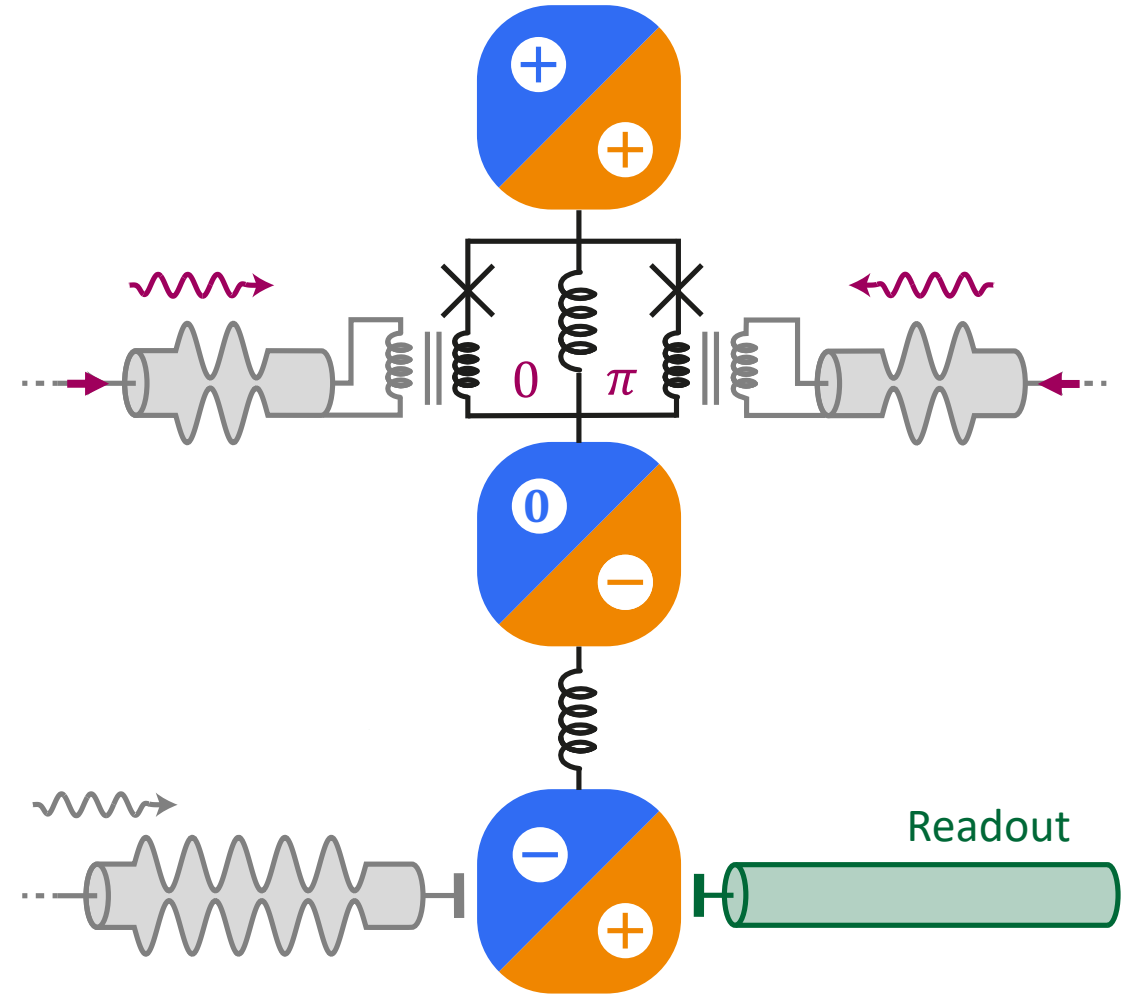
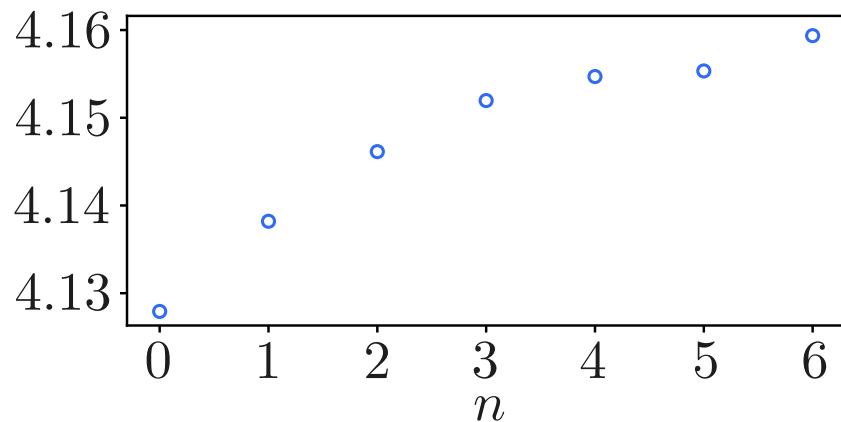

$$\varphi_{ZPF,b} = 0.3$$
$$E_J = h \times 18 \text{ GHz}$$
 DC_1 

Readout

Anharmonicity at the saddle point



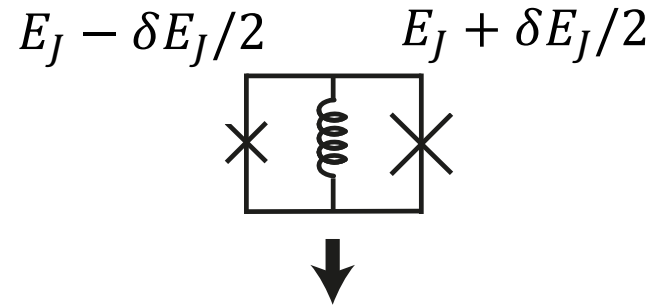
$\omega_a^{n \rightarrow n+1} / 2\pi$ (GHz)



Anharmonicity at the saddle point

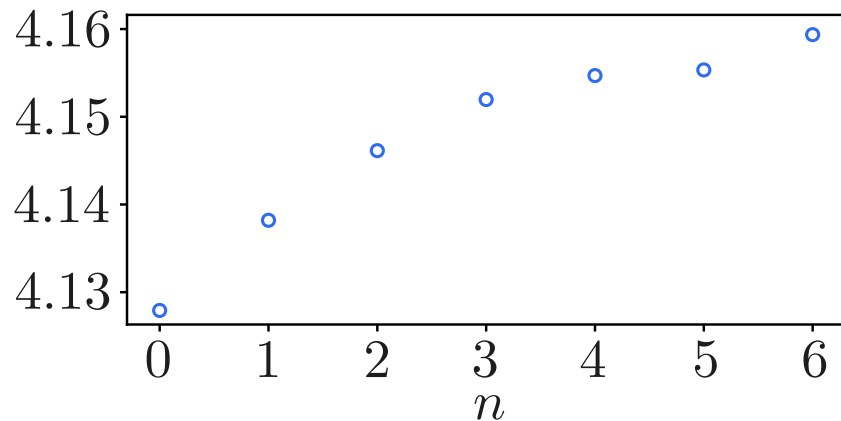
$\vdots$
 $|5\rangle_a$
 $|4\rangle_a$
 $|3\rangle_a$
 $|2\rangle_a$
 $|1\rangle_a$
 $|0\rangle_a$

$$\varphi = \varphi_{ZPF,a}(a + a^\dagger) + \varphi_{ZPF,b}(b + b^\dagger) + \varphi_{ZPF,r}(r + r^\dagger)$$

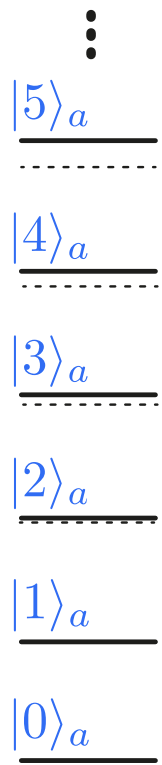


$$H_{asym} = \delta E_J \cos(\varphi)$$

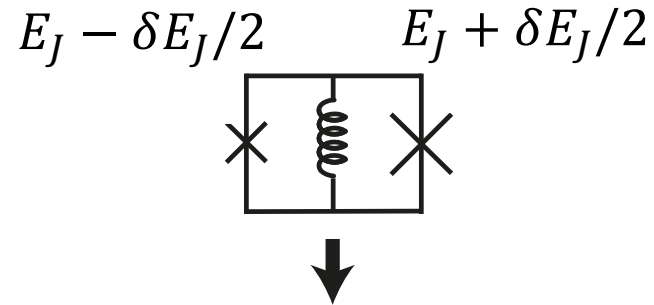
$\omega_a^{n \rightarrow n+1}/2\pi$ (GHz)



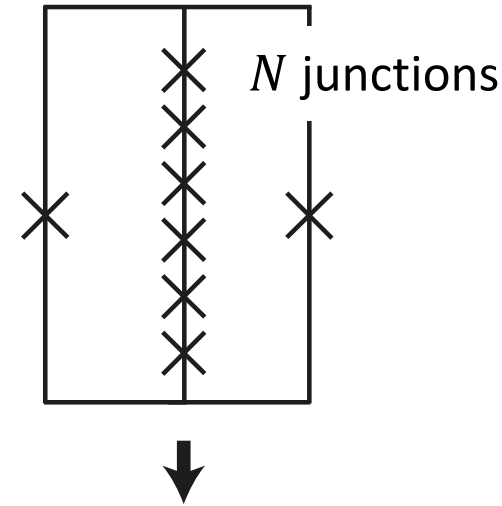
Anharmonicity at the saddle point



$$\varphi = \varphi_{ZPF,a}(a + a^\dagger) + \varphi_{ZPF,b}(b + b^\dagger) + \varphi_{ZPF,r}(r + r^\dagger)$$

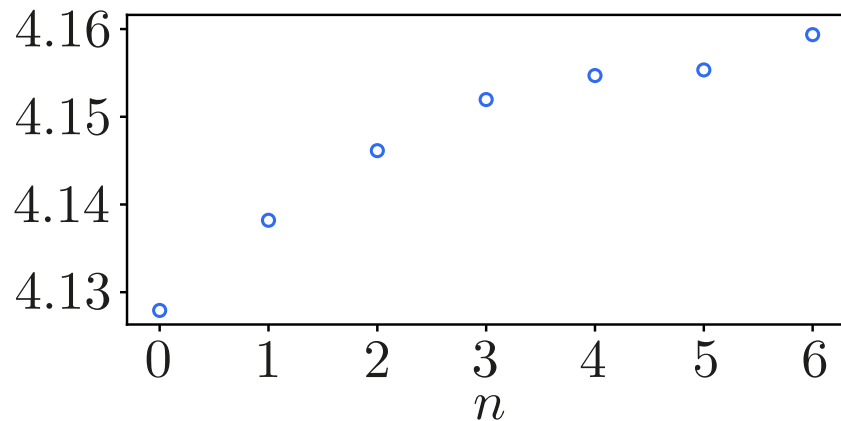


$$H_{asym} = \delta E_J \cos(\varphi)$$



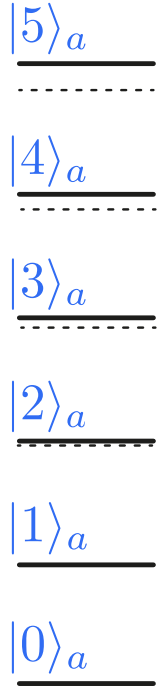
$$H_{chain} = -\frac{E_L}{24 N^2} \varphi^4$$

$\omega_a^{n \rightarrow n+1} / 2\pi$ (GHz)

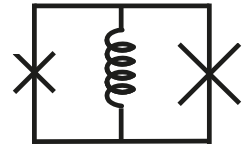


Anharmonicity at the saddle point

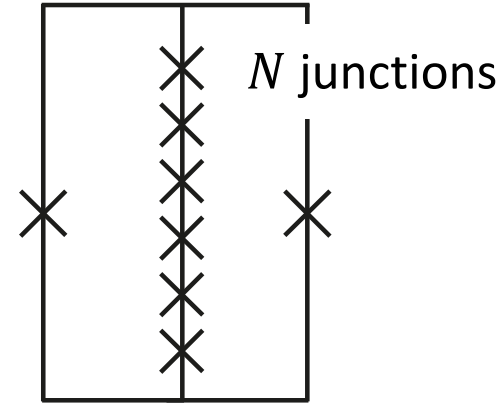
$$\varphi = \varphi_{ZPF,a}(a + a^\dagger) + \varphi_{ZPF,b}(b + b^\dagger) + \varphi_{ZPF,r}(r + r^\dagger)$$



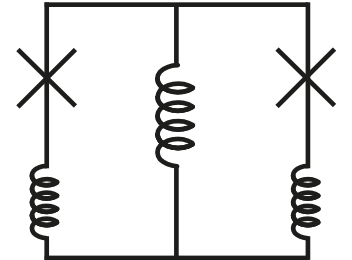
$$E_J - \delta E_J/2 \quad E_J + \delta E_J/2$$



$$H_{\text{asym}} = \delta E_J \cos(\varphi)$$

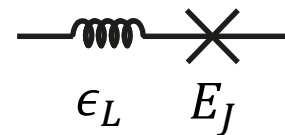
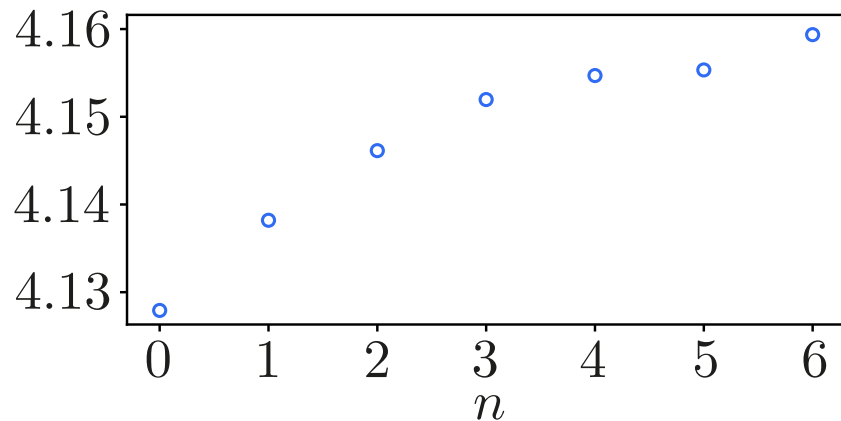


$$H_{\text{chain}} = -\frac{E_L}{24 N^2} \varphi^4$$



$$H_{2\varphi} = \frac{E_J^2}{2\epsilon_L} \cos(2\varphi)$$

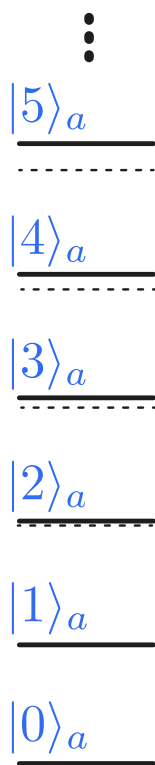
$\omega_a^{n \rightarrow n+1}/2\pi$ (GHz)



$$V_J(\varphi_i) = -E_J \cos(\varphi_i) + \frac{E_J^2}{4\epsilon_L} \cos(2\varphi_i)$$

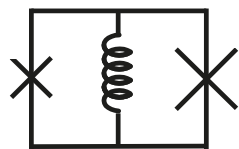
Putterman *et al.*, PRX, 2025
Smith *et al.*, Nat. Comm., 2025

Anharmonicity at the saddle point

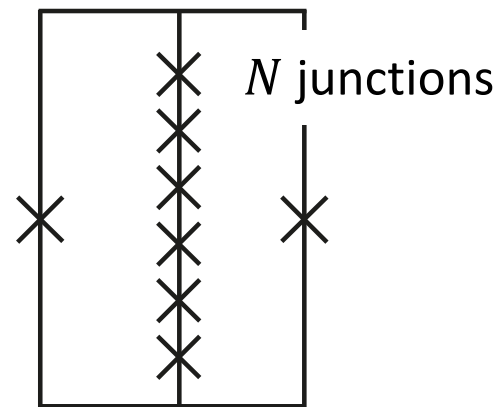


$$\varphi = \varphi_{ZPF,a}(a + a^\dagger) + \varphi_{ZPF,b}(b + b^\dagger) + \varphi_{ZPF,r}(r + r^\dagger)$$

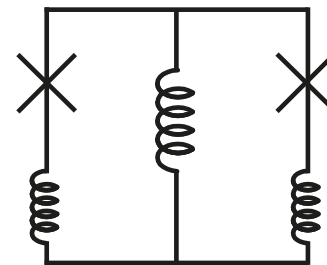
$$E_J - \delta E_J/2 \quad E_J + \delta E_J/2$$



$$H_{\text{asym}} = \delta E_J \cos(\varphi)$$

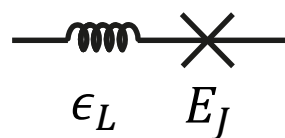
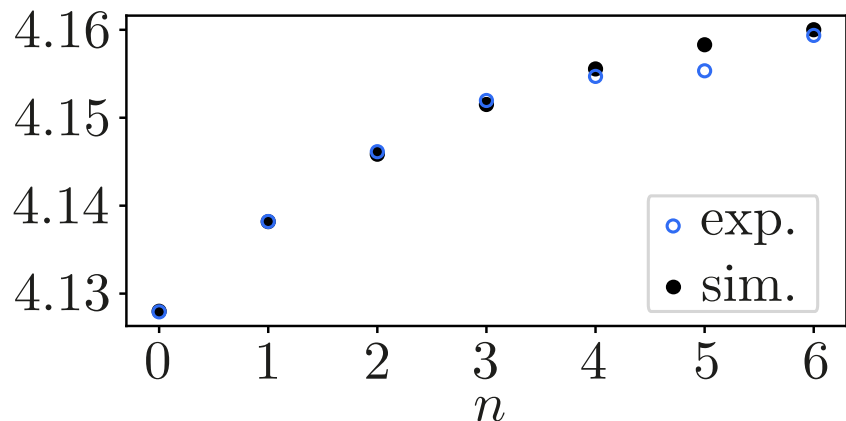


$$H_{\text{chain}} = -\frac{E_L}{24 N^2} \varphi^4$$



$$H_{2\varphi} = \frac{E_J^2}{2\epsilon_L} \cos(2\varphi)$$

$\omega_a^{n \rightarrow n+1}/2\pi$ (GHz)

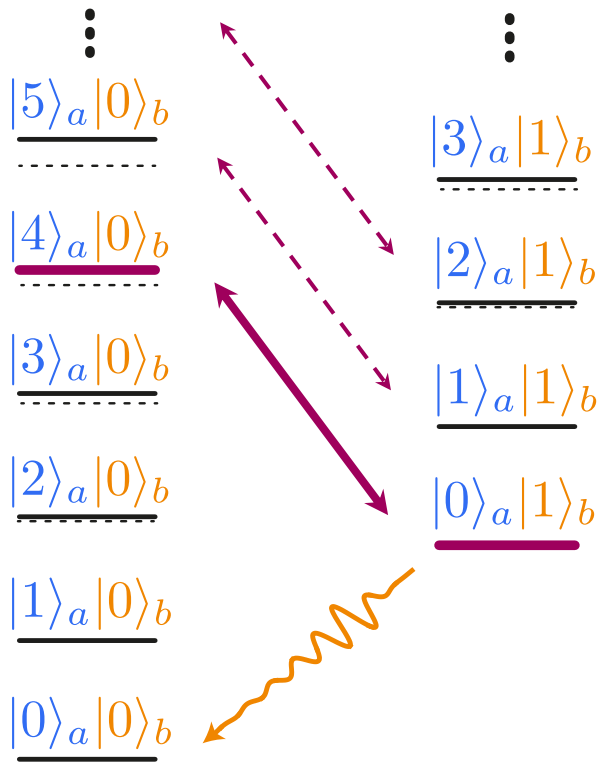


$$\epsilon_L = h \times 2 \text{ THz}$$

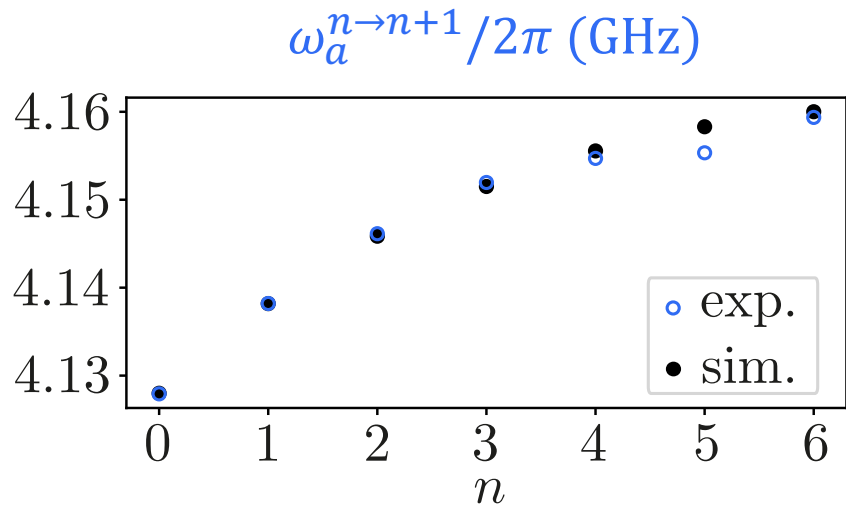
$$V_J(\varphi_i) = -E_J \cos(\varphi_i) + \frac{E_J^2}{4\epsilon_L} \cos(2\varphi_i)$$

$$(E_{2\varphi}/E_\varphi = 0.2 \%)$$

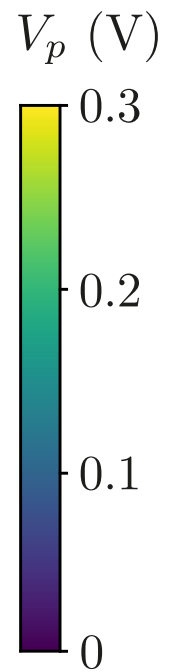
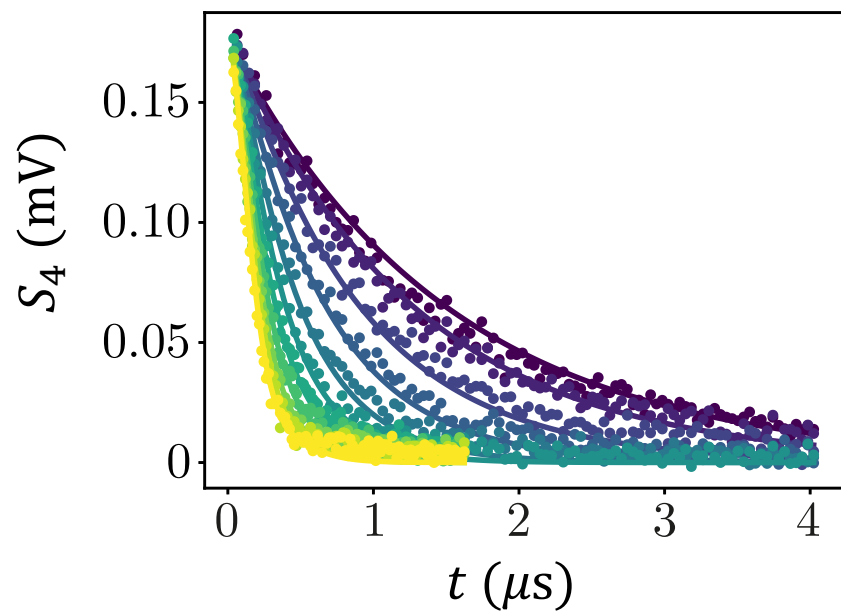
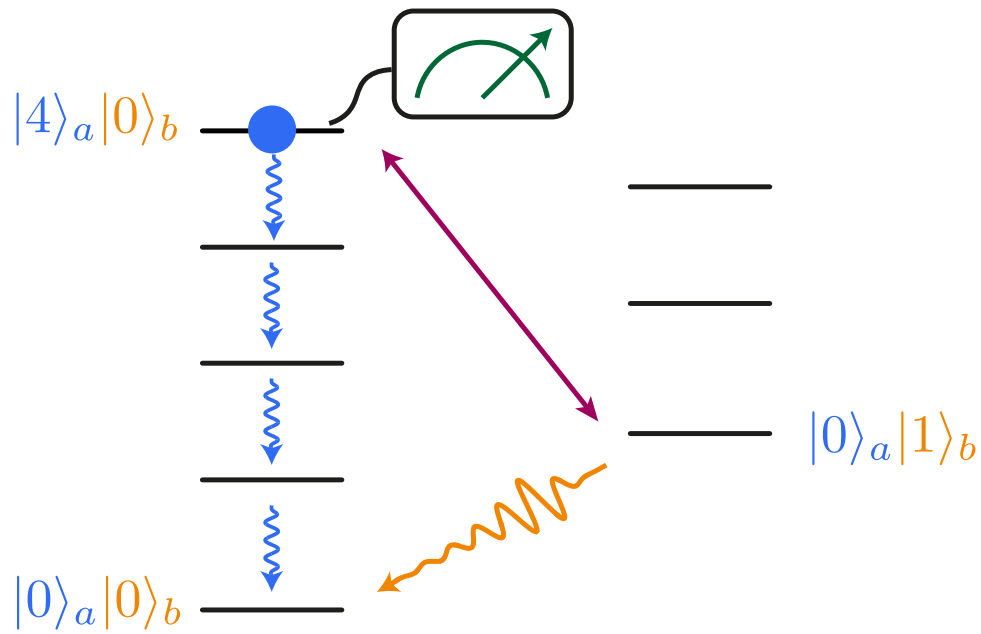
State-dependent dissipation



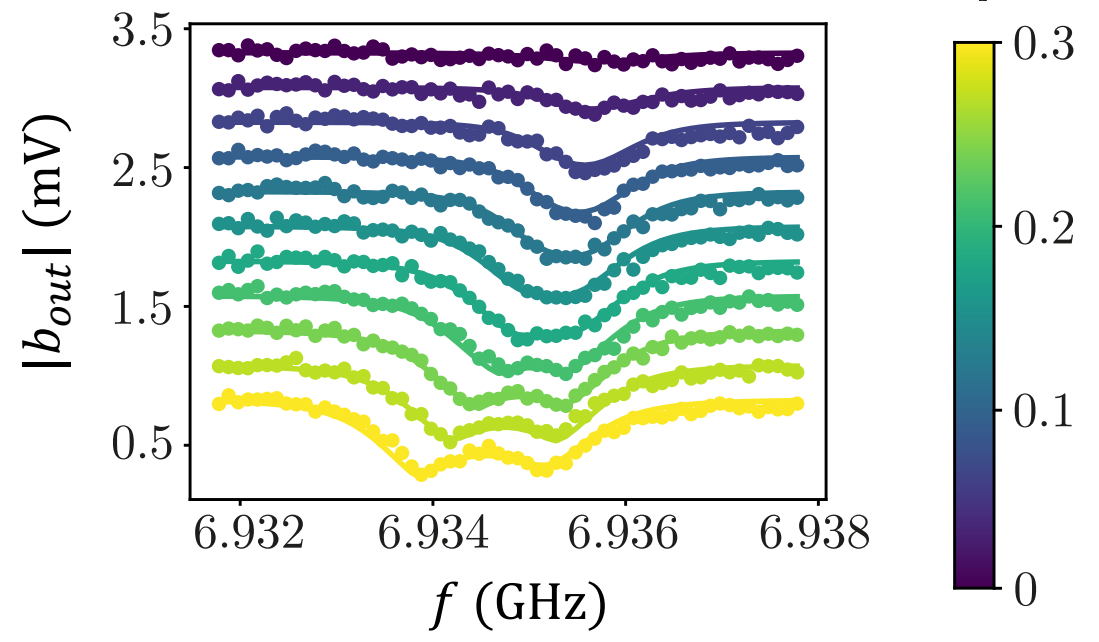
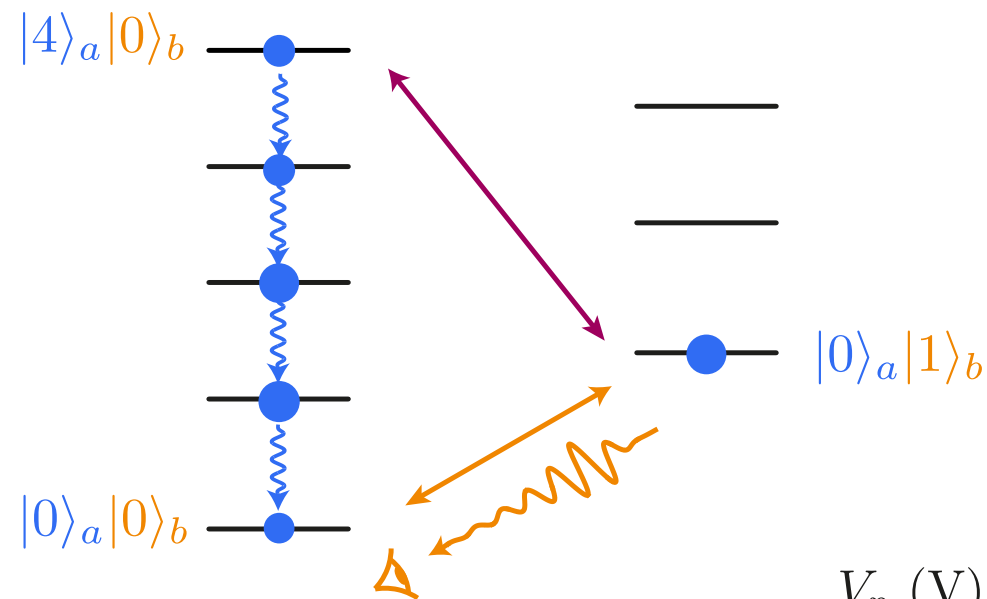
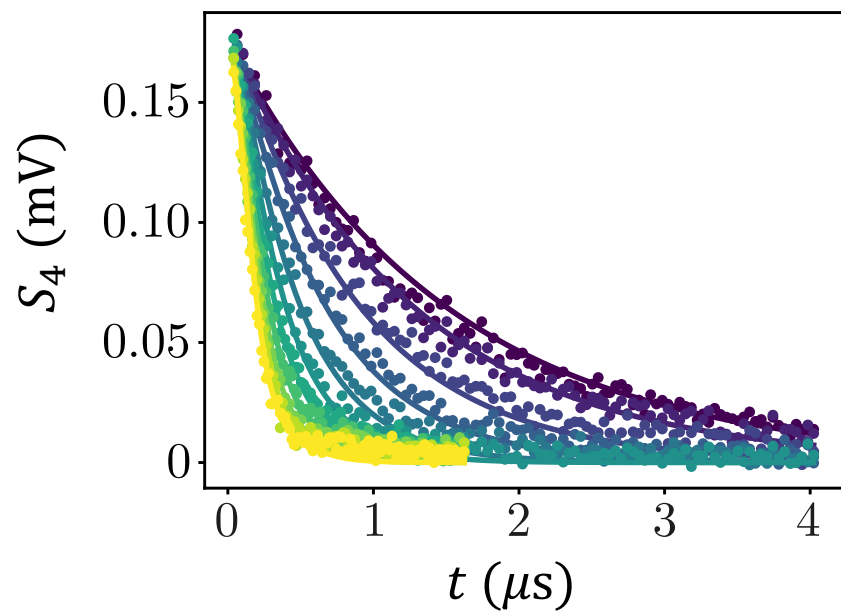
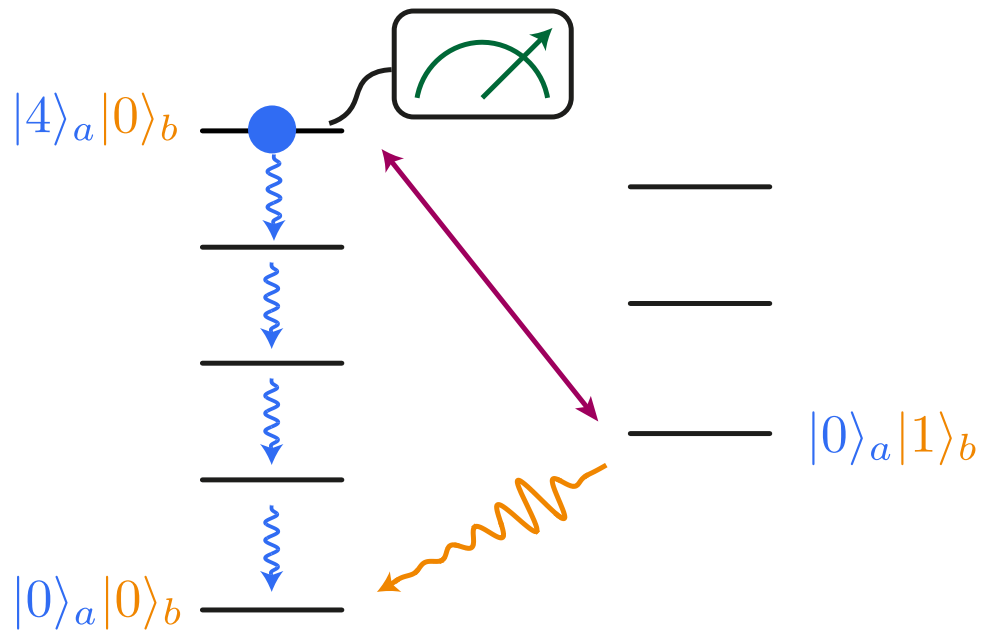
$$\kappa_4 \mathcal{D}[a^4] \longrightarrow \Gamma_4 \mathcal{D}[|0\rangle\langle 4|]$$



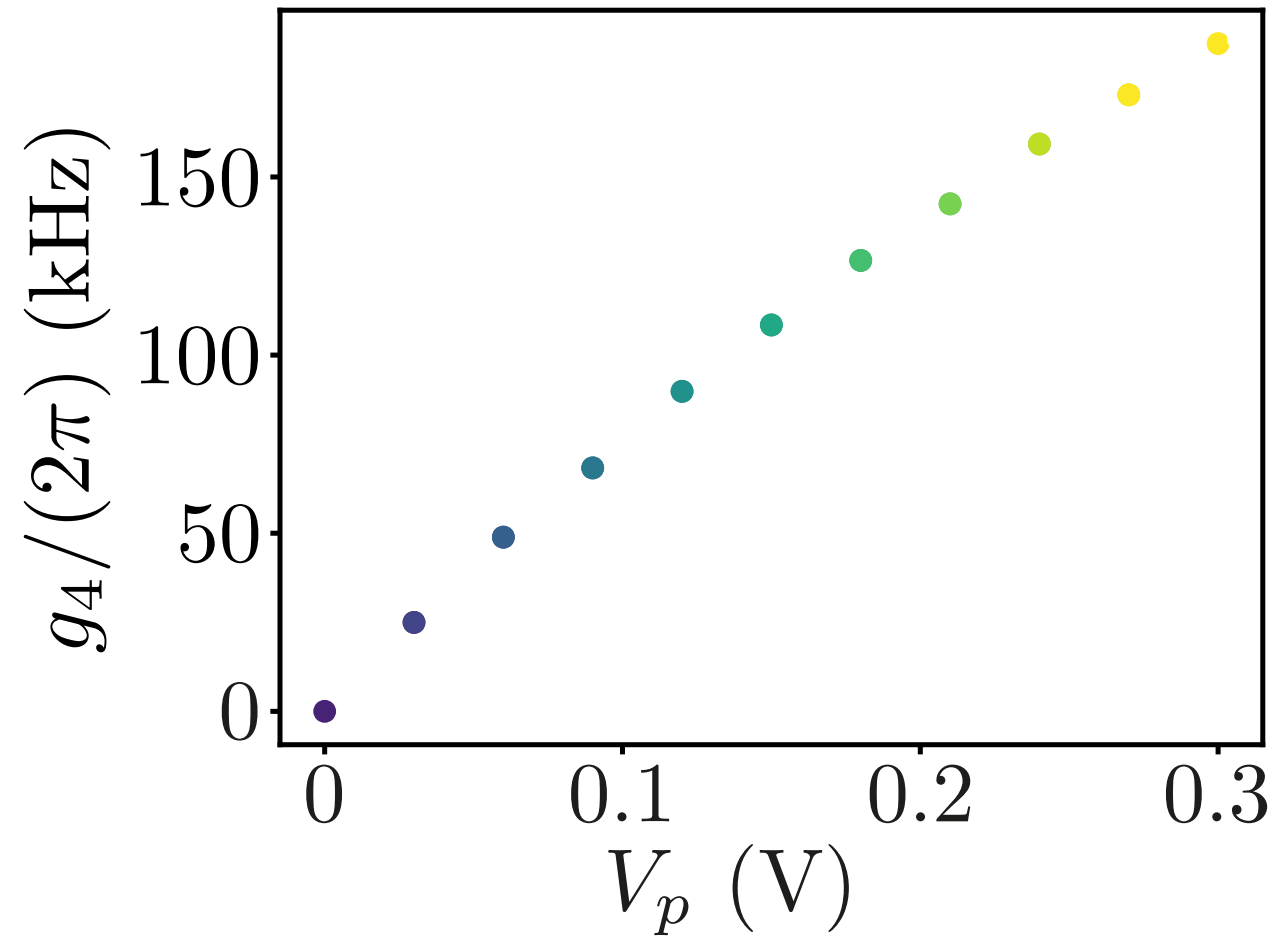
4-to-1 photon exchange



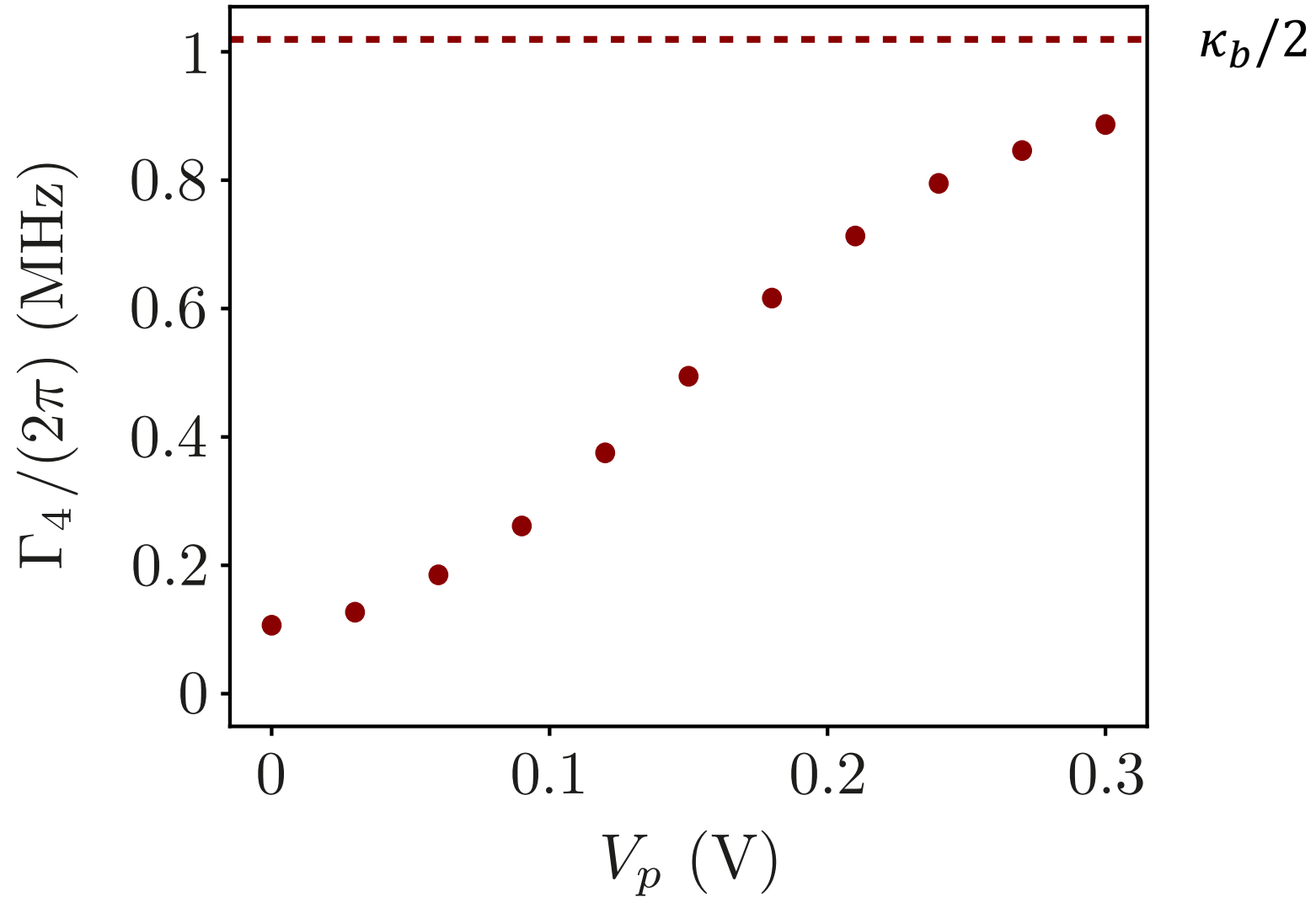
4-to-1 photon exchange



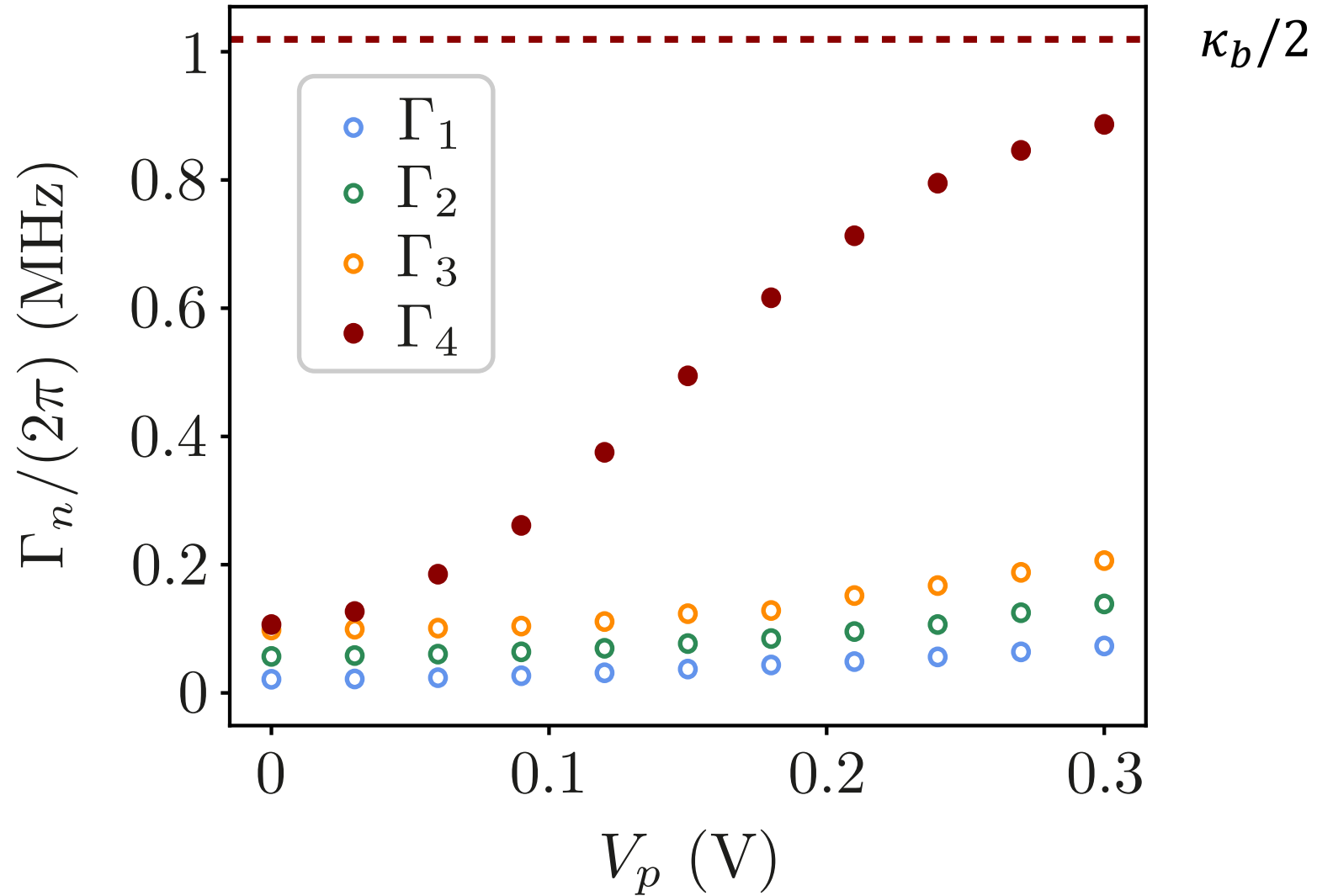
4-to-1 photon exchange



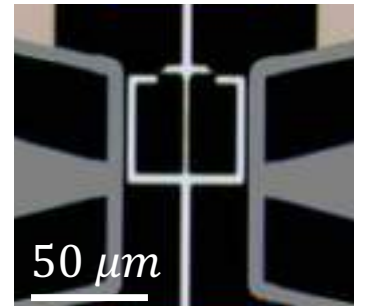
Dissipating quartets of excitations



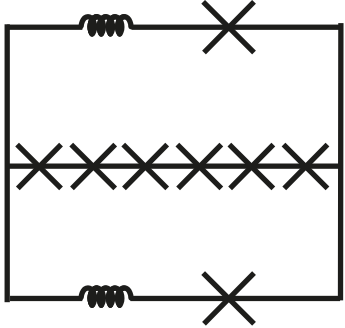
Dissipating quartets of excitations



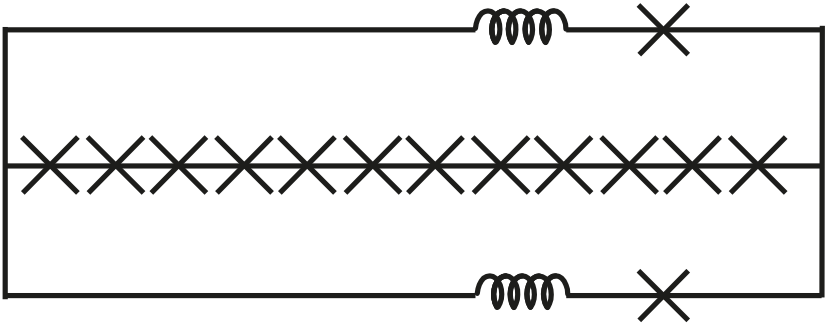
Adapting the ATS's design



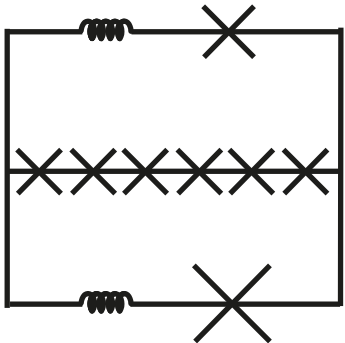
**Too few
junctions**



Too long tracks

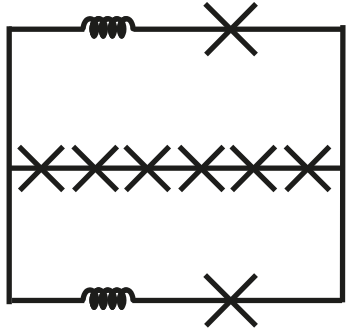


Asymmetry

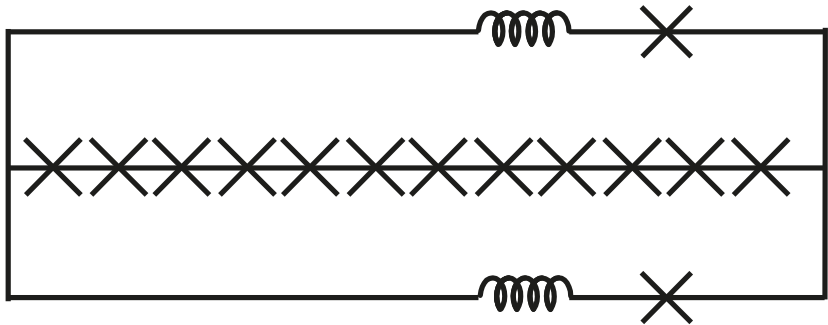


Adapting the ATS's design

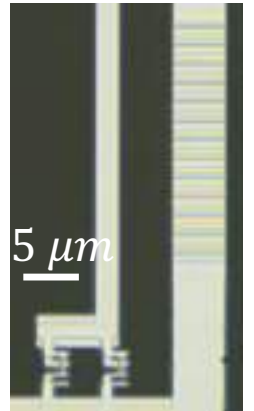
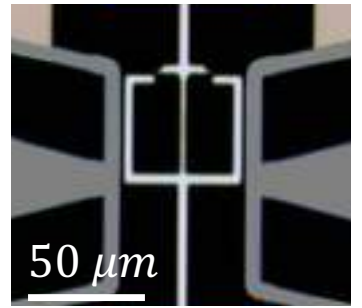
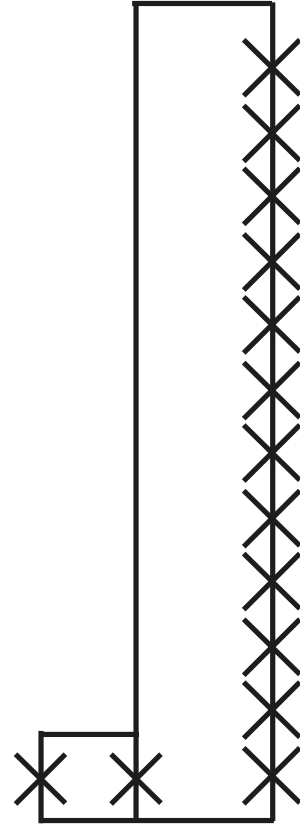
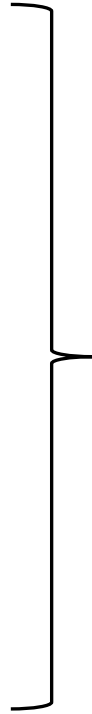
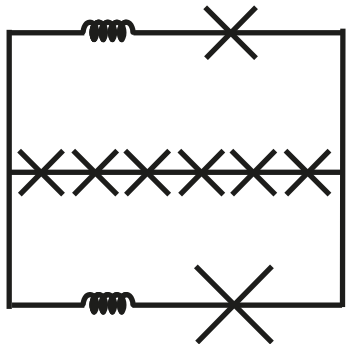
Too few
junctions



Too long tracks

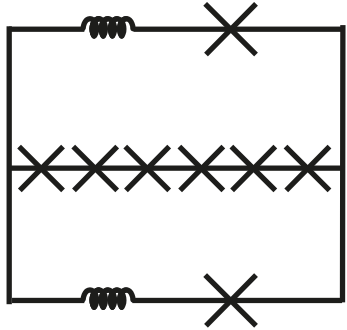


Asymmetry

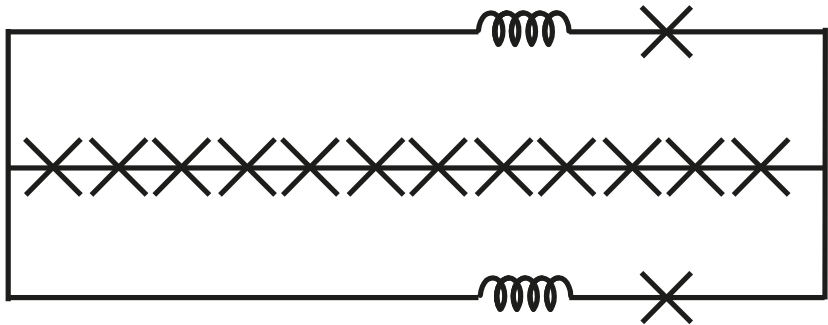


Adapting the ATS's design

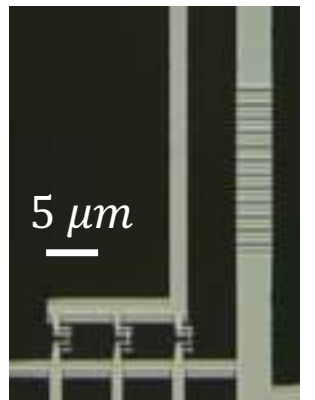
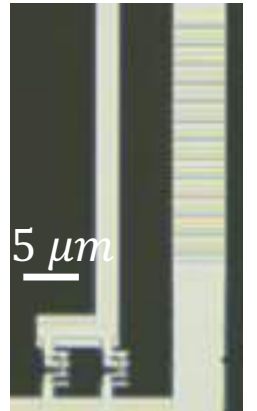
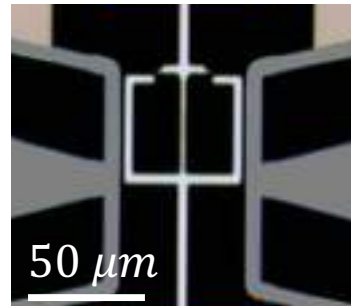
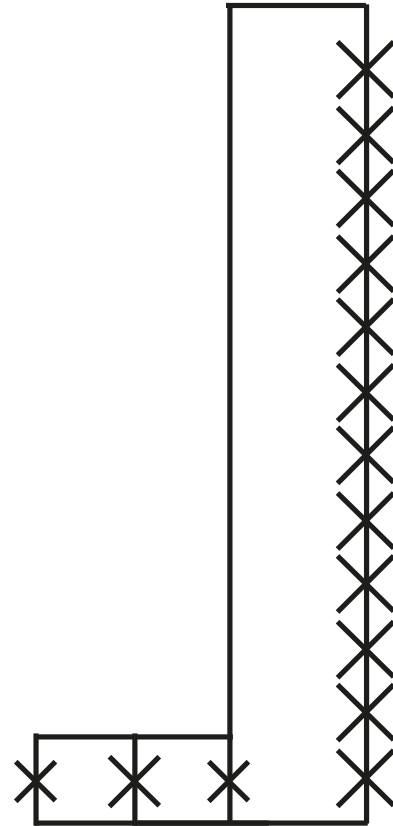
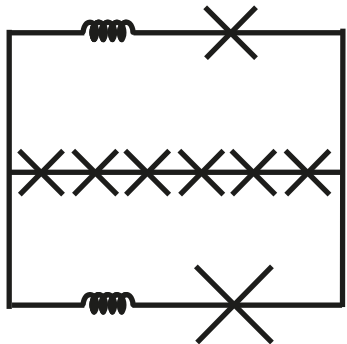
Too few junctions



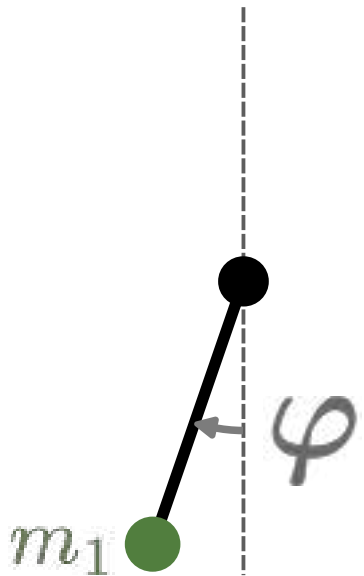
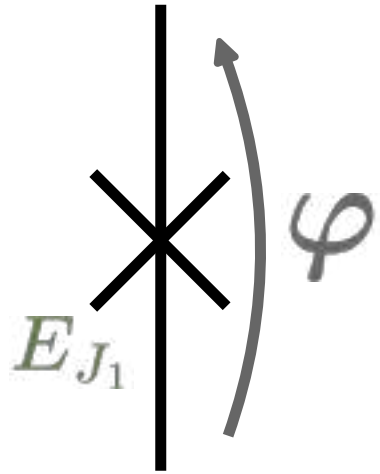
Too long tracks



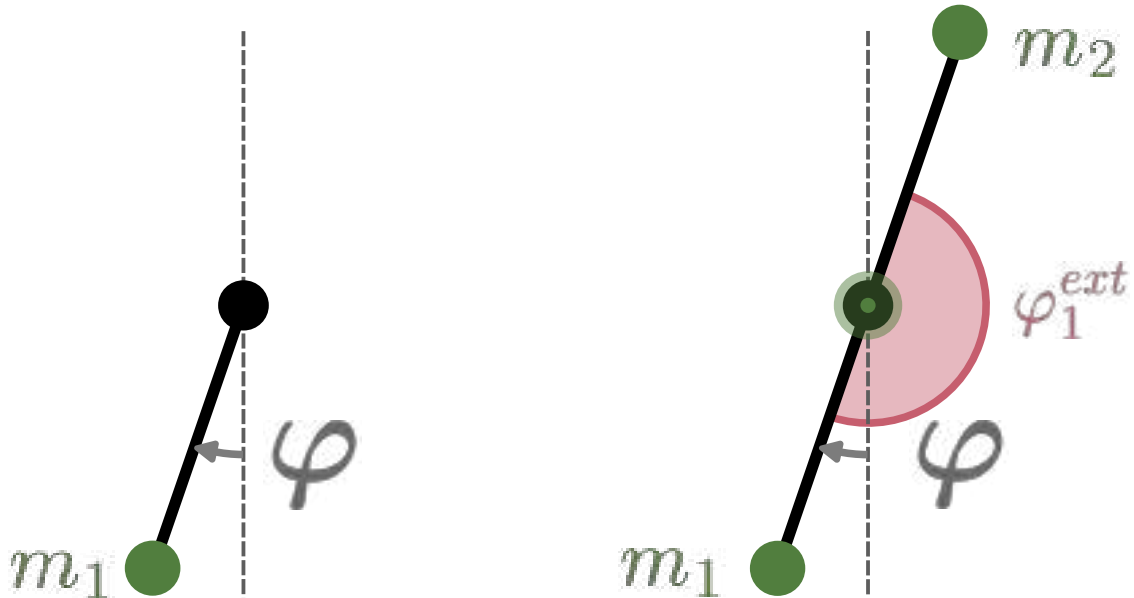
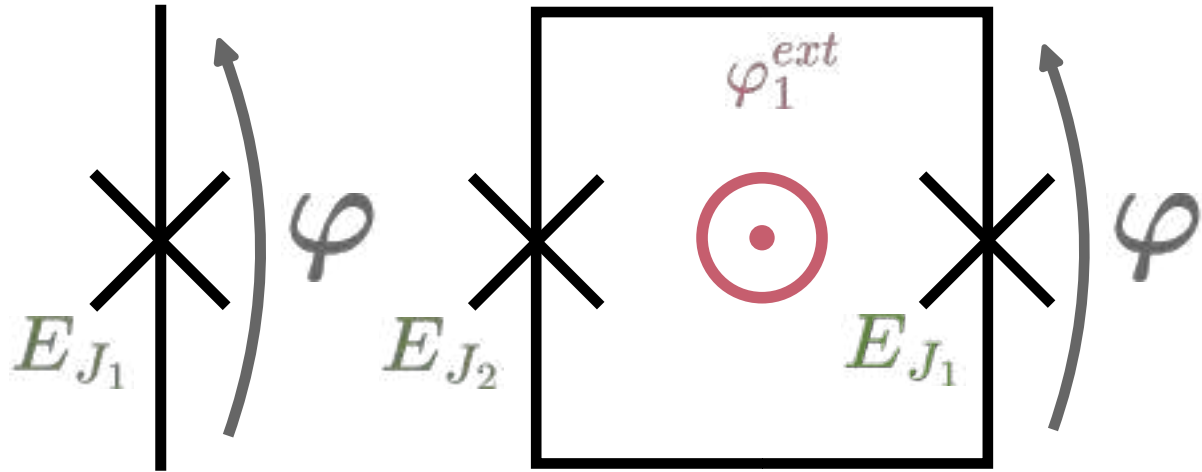
Asymmetry



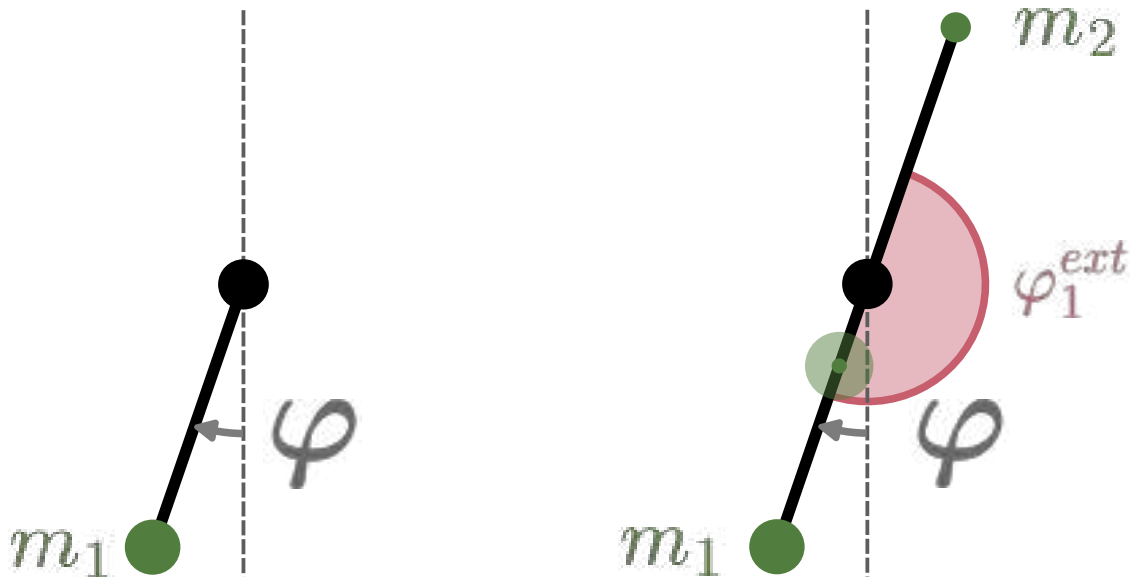
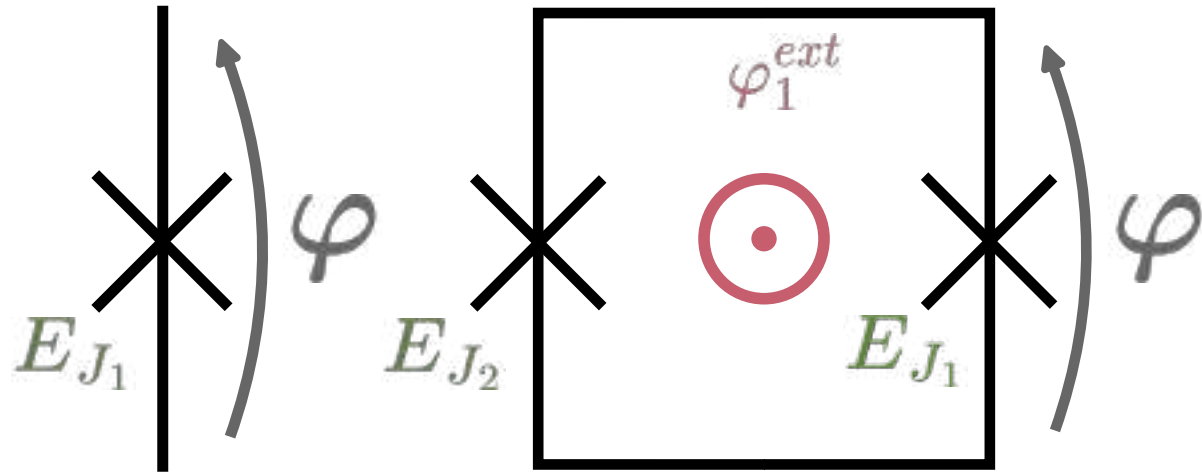
3-SQUID: a mechanical analogy



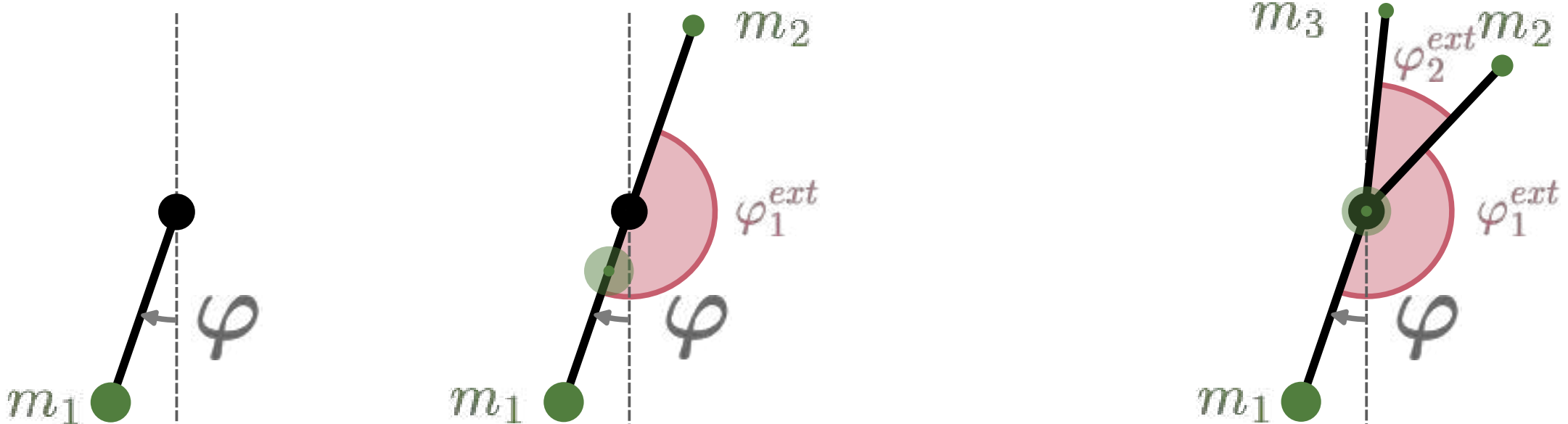
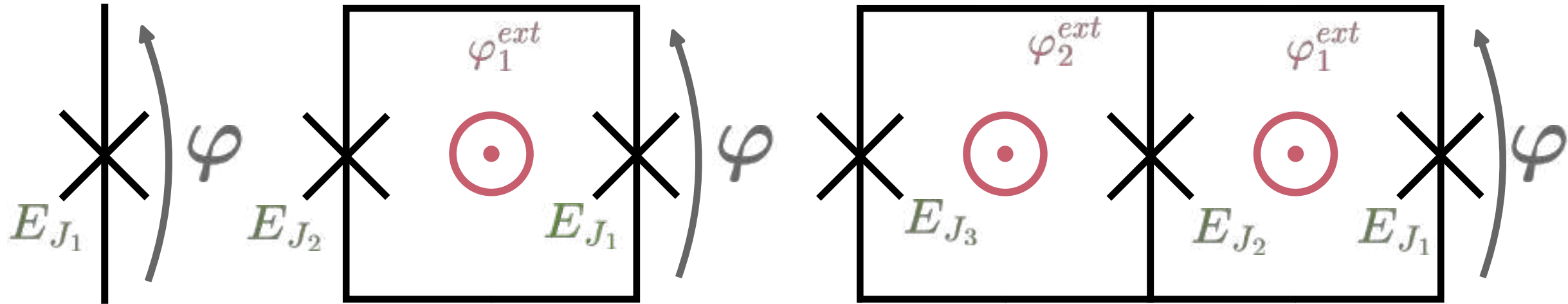
3-SQUID: a mechanical analogy



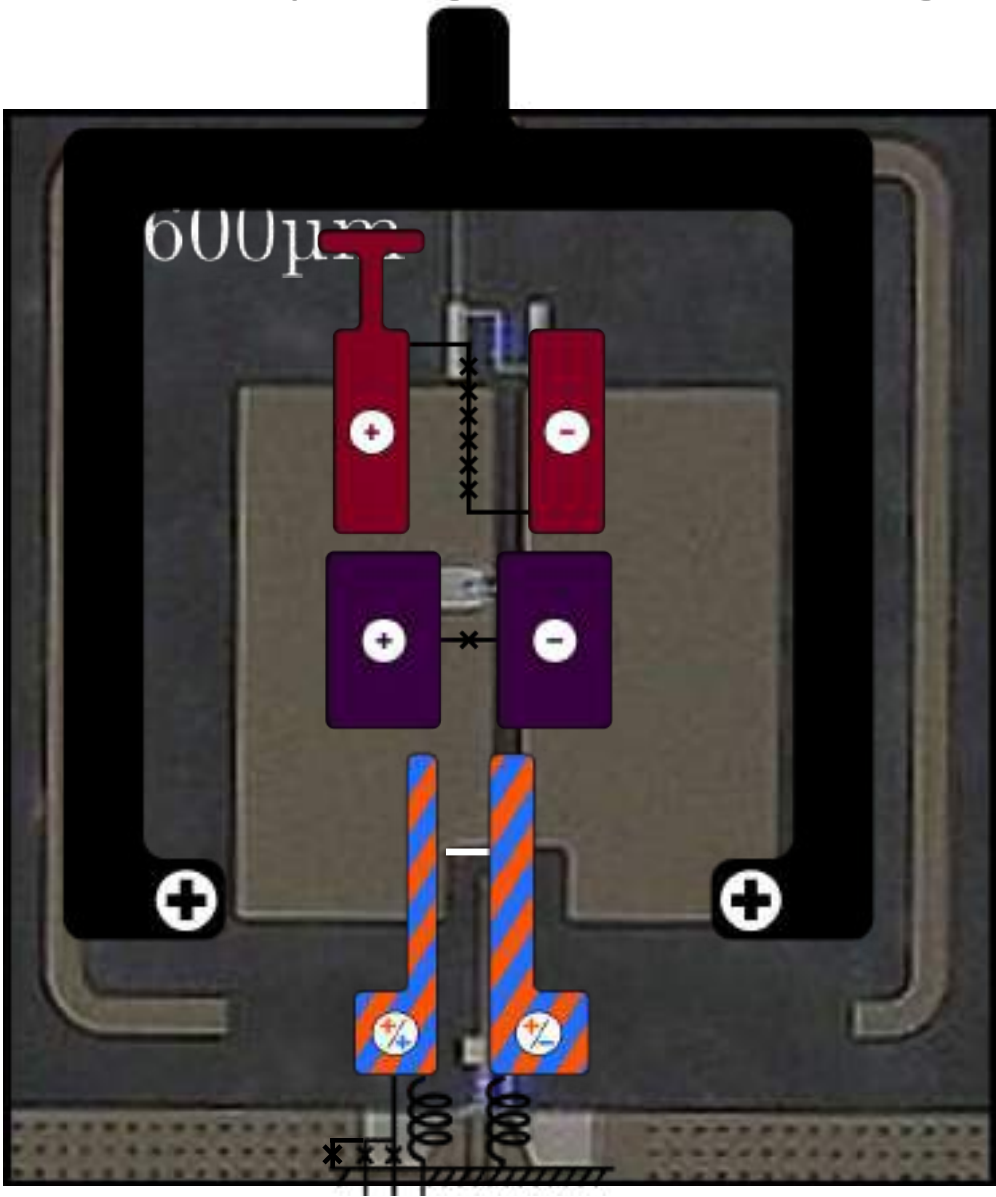
3-SQUID: a mechanical analogy



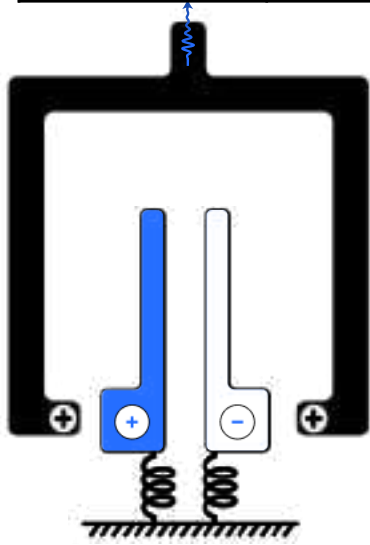
3-SQUID: a mechanical analogy



Adapting circuit design : spatial symmetry

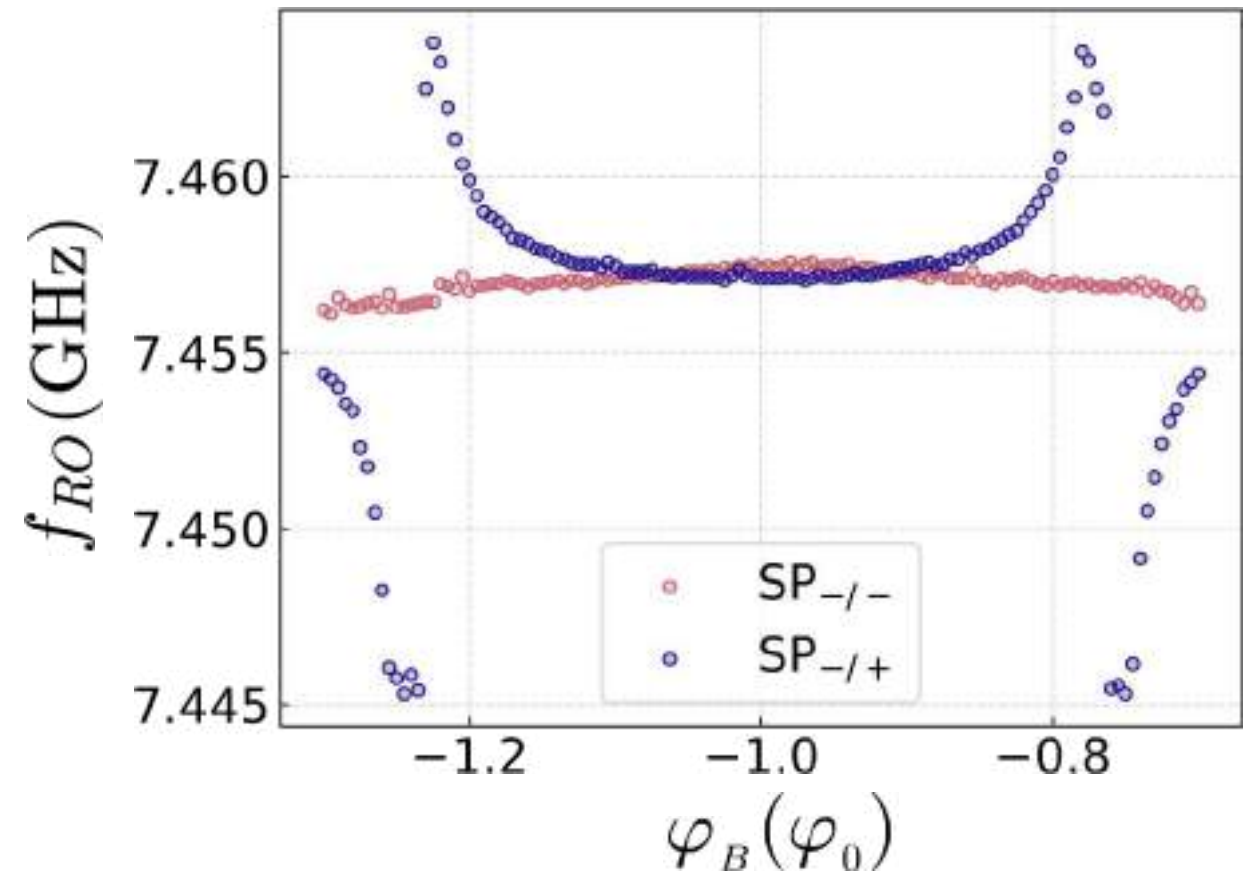
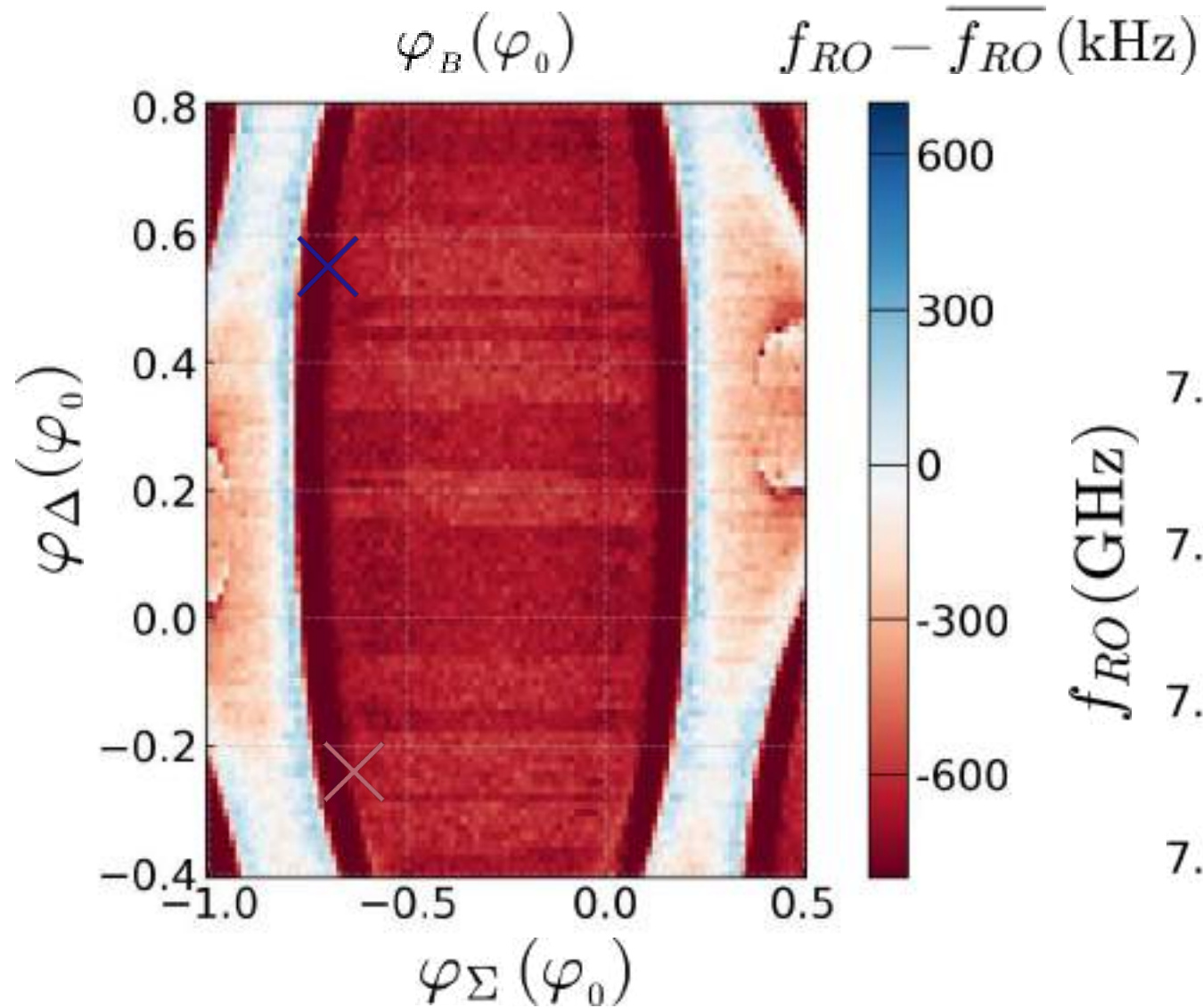


Mode	J [kHz]	$\kappa_{ext}/2\pi$ (MHz)	$\varphi_{ATS}^{ZPF}[\varphi_0]$	$\chi_{tX}/2\pi$ (MHz)
Readout	8	1	0.02	1
Transmon	4	<1e-3	0.02	200
Memory	5	<1e-3	0.45	1
Buffer	7	40	0.45	~0



Preliminary results : 3SQUID

$$\begin{pmatrix} \varphi_{\Sigma} \\ \varphi_{\Delta} \\ \varphi_B \end{pmatrix} = \begin{pmatrix} 1/4 & 1/2 & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \varphi_1^{ext} \\ \varphi_2^{ext} \\ \varphi_3^{ext} \end{pmatrix}$$



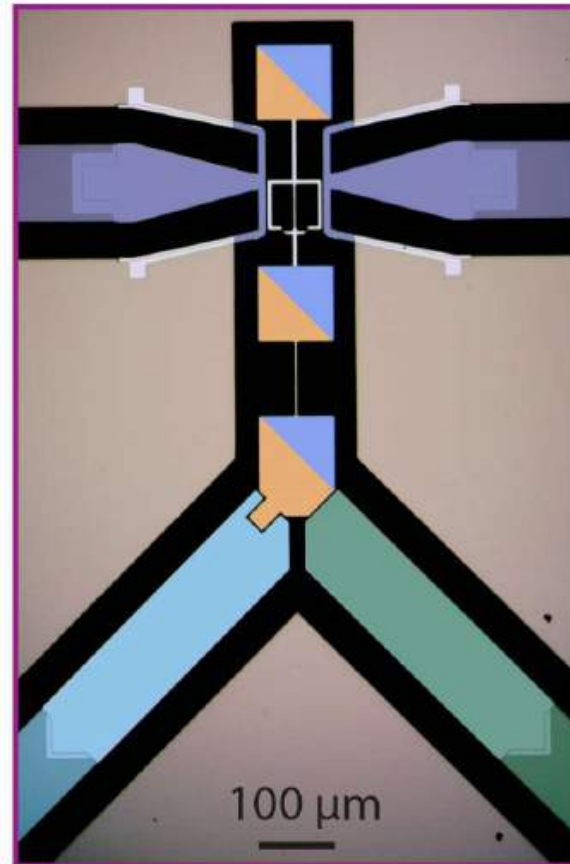
Conclusion

Vanselow *et al.* , PRX, 2026 :

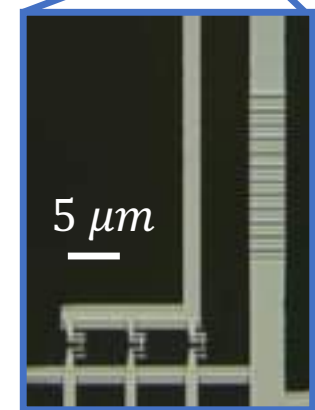
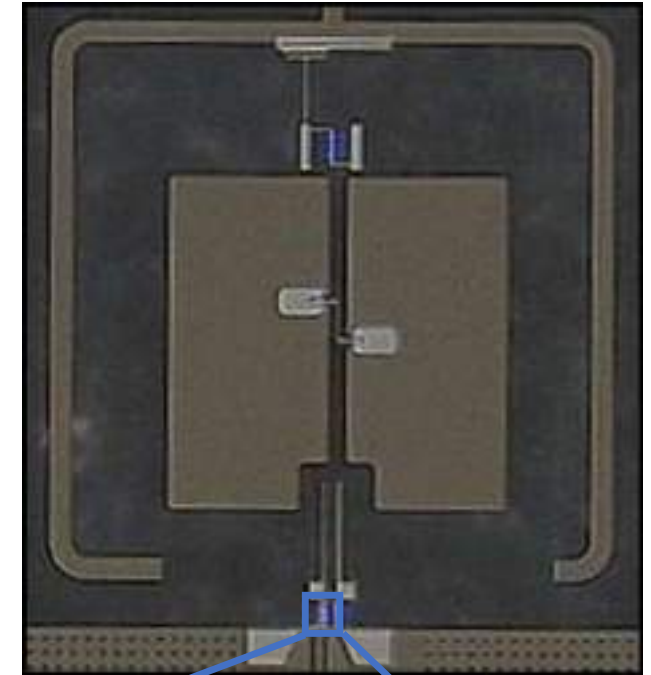
Building a platform for clean
high-order dissipation engineering

Goal : dissipative stabilization of
advanced bosonic qubits

Challenge : Spurious non-linear processes
(anharmonicity, parametric loss)



Current version:



Thank you !



Aron
Vanselow



Lev-Arcady
Sellem



Brieuc
Beauseigneur



Louis
Lattier



Z. Leghtas



M. Mirrahimi



A. Sarlette



R. Robin



Pierre Rouchon